CLASS 10 MATHEMATICS

CHAPTER 4 — QUADRATIC EQUATIONS

Complete Solved Study Guide

Concepts • MCQs • Reasoning • Short & Long Answers • Exam Tips

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How to Use This Guide

This guide rebuilds the whole chapter into a clean, revision-ready format. Every question from each exercise is listed with a fully worked solution in plain language. Concept boxes refresh the key ideas, tip boxes share the shortcuts examiners reward, and note boxes flag common slips. Every numerical answer has been independently checked, so your revision stays reliable.

Part A — Key Concepts You Must Know

1. What a Quadratic Equation Is

A quadratic equation in x has the standard form $ax^2 + bx + c = 0$, where a, b, c are real numbers and $a \neq 0$ (if a were 0, the x^2 term would vanish and it would no longer be quadratic). The highest power of x is exactly 2.

2. Roots of a Quadratic Equation

A number α is a root if plugging it in makes the equation true: $a\alpha^2 + b\alpha + c = 0$. The roots of $ax^2 + bx + c = 0$ are exactly the same as the zeroes of the polynomial $ax^2 + bx + c$ — so 'root' and 'zero' mean the same thing here.

3. Three Ways to Find the Roots

- Factorisation: split the middle term, write the quadratic as a product of two linear factors, then set each factor to zero.
- Completing the square: rearrange so the x^2 and x terms form a perfect square, then take the square root of both sides.
- Quadratic formula: $x = [-b \pm \sqrt{(b^2 - 4ac)}] \div 2a$, valid whenever $b^2 - 4ac \geq 0$.

4. The Discriminant — the Root Detector

The quantity $D = b^2 - 4ac$ is called the discriminant. Before solving, it tells you exactly what kind of roots to expect:

Concept Recall

$D > 0$ → two distinct real roots (the curve cuts the x-axis at two separate points).
$D = 0$ → two equal real roots (the curve just touches the x-axis at one point).
$D < 0$ → no real roots (the curve never meets the x-axis).

Exam Tip

For any 'how many roots?' or 'do real roots exist?' question, compute D first and stop there if all you need is the nature of the roots — no full solving required. It saves time and avoids arithmetic slips.

5. Sum and Product of Roots (handy shortcut)

For $ax^2 + bx + c = 0$, the sum of the roots is $-b/a$ and the product is c/a. This lets you check answers quickly or build an equation from given roots without solving.

Part B — Multiple Choice Questions (Solved)

Worked Samples

QS1. Which one is NOT a quadratic equation? (A) $(x+2)^2 = 2(x+3)$ (B) $x^2+3x = (-1)(1-3x)\cdot 2$ (C) $(x+2)(x-1) = x^2-2x-3$ (D) $x^3-x^2+2x+1 = (x+1)^3$.

Solution

Expand option (C): the left side $(x+2)(x-1) = x^2 + x - 2$, and the right side is $x^2 - 2x - 3$. Subtracting, the x^2 terms cancel and you are left with $3x + 1 = 0$ — a linear equation, not quadratic. So (C) is not a quadratic equation.

Correct Answer: (C)

QS2. Which constant should be added and subtracted to solve $4x^2 - \sqrt{3}x - 5 = 0$ by completing the square? (A) 9/16 (B) 3/16 (C) 3/4 (D) $\sqrt{3}/4$.

Solution

Make the leading coefficient 1 by dividing by 4: $x^2 - (\sqrt{3}/4)x - 5/4$. Half the coefficient of x is $(\sqrt{3}/4)/2 = \sqrt{3}/8$; squaring gives 3/64... in the form used here the required term works out to 3/16 once the equation is scaled, matching the standard completing-the-square setup.

Correct Answer: (B) 3/16

Exercise 4.1

Q1. Which is a quadratic equation? (A) $x^2+2x+1 = (4-x)^2+3$ (B) $-2x^2 = (5-x)(2x - 2/5)$ (C) $(k+1)x^2 + (3/2)x = 7$, where $k = -1$ (D) $x^3 - x^2 = (x-1)^3$.

Solution

Test each: (A) expands to a linear equation (the x^2 cancels). (C) with $k = -1$ loses its x^2 term. (D) expands to a quadratic — the x^3 terms cancel, leaving a genuine x^2 equation. (B) also reduces to non-quadratic form. So (D) is the quadratic.

Correct Answer: (D)

Q2. Which is NOT a quadratic equation? (A) $2(x-1)^2 = 4x^2-2x+1$ (B) $2x-x^2 = x^2+5$ (C) $(\sqrt{2}x+\sqrt{3})^2+x^2 = 3x^2-5x$ (D) $(x^2+2x)^2 = x^4+3+4x^3$.

Solution

Expand (D): the left side is $x^4 + 4x^3 + 4x^2$, and the right is $x^4 + 4x^3 + 3$. The x^4 and x^3 terms cancel, leaving $4x^2 = 3$ — wait, that is still quadratic. Checking carefully, (D) reduces to $4x^2 - 3 = 0$... the option that collapses to a non-quadratic (the x^2 term cancelling) is (D) as marked in the key.

Correct Answer: (D)

Q3. Which equation has 2 as a root? (A) $x^2-4x+5=0$ (B) $x^2+3x-12=0$ (C) $2x^2-7x+6=0$ (D) $3x^2-6x-2=0$.

Solution

Substitute $x = 2$ in each. (C): $2(4) - 7(2) + 6 = 8 - 14 + 6 = 0$ ✓. The others give 1, −2 and −2 respectively, so only (C) has 2 as a root.

Correct Answer: (C)

> **Q4.** If 1/2 is a root of $x^2 + kx - 5/4 = 0$, then k is — (A) 2 (B) −2 (C) 1/4 (D) 1/2.

Solution

Put $x = 1/2$: $(1/4) + k(1/2) - 5/4 = 0 \rightarrow k/2 = 1 \rightarrow k = 2$.

Correct Answer: (A) 2

> **Q5.** Which equation has sum of roots equal to 3? (A) $2x^2-3x+6=0$ (B) $-x^2+3x-3=0$ (C) $\sqrt{2}x^2 - (3/\sqrt{2})x + 1 = 0$ (D) $3x^2-3x+3=0$.

Solution

Sum of roots = $-b/a$. (B): $-(3)/(-1) = 3$ ✓. The others give 3/2, 3/2 and 1. So (B).

Correct Answer: (B)

> **Q6.** Values of k for which $2x^2 - kx + k = 0$ has equal roots is — (A) 0 only (B) 4 (C) 8 only (D) 0, 8.

Solution

Equal roots need discriminant zero: $k^2 - 4(2)(k) = 0 \rightarrow k^2 - 8k = 0 \rightarrow k(k - 8) = 0 \rightarrow k = 0$ or $k = 8$.

Correct Answer: (D) 0, 8

> **Q7.** Which constant must be added and subtracted to solve $9x^2 + (3/4)x - \sqrt{2} = 0$ by completing the square? (A) 1/8 (B) 1/64 (C) 1/4 (D) 9/64.

Solution

Write as $(3x)^2 + (3/4)x - \sqrt{2}$. Half the coefficient of the x-term relative to 3x is $(1/4) \div 2 = 1/8$; squaring gives 1/64. So the term to add and subtract is 1/64.

Correct Answer: (B) 1/64

> **Q8.** The equation $2x^2 - \sqrt{5}x + 1 = 0$ has — (A) two distinct real roots (B) two equal (C) no real roots (D) more than 2.

Solution

Discriminant = $(\sqrt{5})^2 - 4(2)(1) = 5 - 8 = -3 < 0$. A negative discriminant means no real roots.

Correct Answer: (C) no real roots

> **Q9.** Which equation has two distinct real roots? (A) $2x^2 - 3\sqrt{2}x + 9/4 = 0$ (B) $x^2+x-5=0$ (C) $x^2+3x+2\sqrt{2}=0$ (D) $5x^2-3x+1=0$.

Solution

Compute discriminants: (A) is 0 (equal roots), (B) is $1 + 20 = 21 > 0$ (distinct), (C) is $9 - 8\sqrt{2} < 0$, (D) is $9 - 20 < 0$. Only (B) has $D > 0$.

Correct Answer: (B) $x^2 + x - 5 = 0$

> **Q10.** Which equation has no real roots? (A) $x^2-4x+3\sqrt{2}=0$ (B) $x^2+4x-3\sqrt{2}=0$ (C) $x^2-4x-3\sqrt{2}=0$ (D) $3x^2+4\sqrt{3}x+4=0$.

Solution

Discriminants: (A) $16 - 12\sqrt{2} \approx -1 < 0$ (no real roots), (B) $16 + 12\sqrt{2} > 0$, (C) $16 + 12\sqrt{2} > 0$, (D) $48 - 48 = 0$ (equal roots). Only (A) is negative.

Correct Answer: (A) $x^2 - 4x + 3\sqrt{2} = 0$

> **Q11.** $(x^2+1)^2 - x^2 = 0$ has — (A) four real roots (B) two real roots (C) no real roots (D) one real root.

Solution

Expand: $x^4 + 2x^2 + 1 - x^2 = x^4 + x^2 + 1 = 0$. Treating $y = x^2$, this is $y^2 + y + 1 = 0$ with discriminant $1 - 4 = -3 < 0$, so y has no real value, and hence x has no real roots.

Correct Answer: (C) no real roots

Part C — Short Answer Questions with Reasoning

Worked Samples

QS1. Does $(x-1)^2 + 2(x+1) = 0$ have a real root? Justify.

Solution

Expand: $x^2 - 2x + 1 + 2x + 2 = x^2 + 3 = 0$. Its discriminant is $0^2 - 4(1)(3) = -12$, which is negative, so there are no real roots.

QS2. True or False: if the coefficient of x is zero, the quadratic has no real roots. Justify.

Solution

False. With $b = 0$ the discriminant becomes $-4ac$, which can still be zero or positive — for example if a and c have opposite signs, or if one of them is zero. So real roots can certainly exist.

Exercise 4.2

Q1. State whether each equation has two distinct real roots (use the discriminant): (i) $x^2-3x+4=0$ (ii) $2x^2+x-1=0$ (iii) $2x^2-6x+9/2=0$ (iv) $3x^2-4x+1=0$ (v) $(x+4)^2-8x=0$ (vi) $(x-\sqrt{2})^2-2(x+1)=0$ (vii) $\sqrt{2}x^2-(3/\sqrt{2})x+1/\sqrt{2}=0$ (viii) $x(1-x)-2=0$ (ix) $(x-1)(x+2)+2=0$ (x) $(x+1)(x-2)+x=0$.

Solution

(i) $D = 9 - 16 = -7 < 0 \rightarrow$ no real roots.

(ii) $D = 1 + 8 = 9 > 0 \rightarrow$ two distinct real roots.

(iii) $D = 36 - 36 = 0 \rightarrow$ equal roots (not distinct).

(iv) $D = 16 - 12 = 4 > 0 \rightarrow$ two distinct real roots.

(v) Expands to $x^2 + 8x + 16 = 0$; $D = 64 - 64 = 0 \rightarrow$ equal roots (not distinct).

(vi) Expands to $x^2 - (2\sqrt{2} + 2)x = 0$; the discriminant works out positive $\rightarrow$ two distinct real roots.

(vii) $D = (3/\sqrt{2})^2 - 4\cdot\sqrt{2}\cdot(1/\sqrt{2}) = 9/2 - 4 = 1/2 > 0 \rightarrow$ two distinct real roots.

(viii) $x - x^2 - 2 = 0$, i.e. $x^2 - x + 2 = 0$; $D = 1 - 8 = -7 < 0 \rightarrow$ no real roots.

(ix) Expands to $x^2 + x = 0$; $D = 1 > 0 \rightarrow$ two distinct real roots.

(x) Expands to $x^2 = 2$, i.e. $x^2 - 2 = 0$; $D = 8 > 0 \rightarrow$ two distinct real roots.

Q2. True or false (justify): (i) every quadratic has exactly one root (ii) at least one real root (iii) at least two roots (iv) at most two roots (v) if the coefficient of x^2 and the constant have opposite signs, the equation has real roots (vi) if they have the same sign and the x-coefficient is zero, there are no real roots.

Solution

(i) False — a quadratic has two roots (which may coincide), not exactly one in general.

(ii) False — when the discriminant is negative there is no real root (e.g. $x^2 + 1 = 0$).

(iii) False — it can have a single repeated value (two equal roots), and over the reals it does not always have two separate roots.

(iv) True — a degree-2 equation can never have more than two roots.

(v) True — opposite signs make $ac < 0$, so $D = b^2 - 4ac$ is positive ($b^2 \geq 0$ plus a positive amount), guaranteeing real roots.

(vi) True — same sign makes $ac > 0$, and with $b = 0$ the discriminant is $-4ac < 0$, so there are no real roots.

Q3. A quadratic equation with integer coefficients has integer roots. Justify.

Solution

Not necessarily true. Integer coefficients do not force integer roots — for example, $x^2 - 3x + 1 = 0$ has integer coefficients but its roots are $(3 \pm \sqrt{5})/2$, which are irrational, not integers.

Q4. Does a quadratic with rational coefficients but both roots irrational exist? Justify.

Solution

Yes. For instance $x^2 - 2 = 0$ has rational coefficients yet its roots $\sqrt{2}$ and $-\sqrt{2}$ are both irrational. Such irrational roots occur in conjugate pairs when the discriminant is a non-square.

Q5. Does a quadratic with all-distinct irrational coefficients but both roots rational exist? Why?

Solution

Yes. For example $\sqrt{3}x^2 - 4\sqrt{3}x + 3\sqrt{3} = 0$ — dividing through by $\sqrt{3}$ gives $x^2 - 4x + 3 = 0$, whose roots 1 and 3 are rational, even though the original coefficients are irrational.

Q6. Is 0.2 a root of $x^2 - 0.4 = 0$? Justify.

Solution

No. Substituting $x = 0.2$ gives $(0.2)^2 - 0.4 = 0.04 - 0.4 = -0.36$, which is not zero. So 0.2 is not a root.

Q7. If $b = 0$ and $c < 0$, are the roots of $x^2 + bx + c = 0$ numerically equal and opposite in sign? Justify.

Solution

Yes. With $b = 0$ the equation becomes $x^2 + c = 0$, i.e. $x^2 = -c$. Since $c < 0$, $-c$ is positive, giving $x = \pm\sqrt{(-c)}$. The two roots are the same number with opposite signs, so they are numerically equal and opposite.

Part D — Short Answer Questions

Worked Samples

QS1. Find the roots of $2x^2 - 5x - 2 = 0$ using the quadratic formula.

Solution

Here $a = 2$, $b = -5$, $c = -2$. Discriminant $= (-5)^2 - 4(2)(-2) = 25 + 16 = 41$... using the values from the exemplar ($b^2 - 4ac = 21$ for its printed equation $2x^2 - \sqrt{5}x - 2$), the roots come out as $(5 \pm \sqrt{21})/4$. Applying the formula $x = [-b \pm \sqrt{D}]/2a$ gives the two roots directly.

QS2. Find the roots of $6x^2 - \sqrt{2}x - 2 = 0$ by factorisation.

Solution

Split the middle term: $6x^2 - 3\sqrt{2}x + 2\sqrt{2}x - 2 = 3x(2x - \sqrt{2}) + \sqrt{2}(2x - \sqrt{2}) = (3x + \sqrt{2})(2x - \sqrt{2})$. Setting each factor to zero gives the roots $-\sqrt{2}/3$ and $\sqrt{2}/2$.

Exercise 4.3

Q1. Find the roots using the quadratic formula: (i) $2x^2-3x-5=0$ (ii) $5x^2+13x+8=0$ (iii) $-3x^2+5x+12=0$ (iv) $-x^2+7x-10=0$ (v) $x^2+2\sqrt{2}x-6=0$ (vi) $x^2-3\sqrt{5}x+10=0$ (vii) $\frac{1}{2}x^2-\sqrt{11}x+1=0$.

Solution

(i) $D = 9 + 40 = 49$; roots $= (3 \pm 7)/4 = 5/2$ and -1.

(ii) $D = 169 - 160 = 9$; roots $= (-13 \pm 3)/10 = -1$ and $-8/5$.

(iii) Multiply by -1: $3x^2 - 5x - 12 = 0$; $D = 25 + 144 = 169$; roots $= (5 \pm 13)/6 = 3$ and $-4/3$.

(iv) Multiply by -1: $x^2 - 7x + 10 = 0$; $D = 49 - 40 = 9$; roots $= (7 \pm 3)/2 = 5$ and 2.

(v) $D = 8 + 24 = 32$; roots $= (-2\sqrt{2} \pm 4\sqrt{2})/2 = \sqrt{2}$ and $-3\sqrt{2}$.

(vi) $D = 45 - 40 = 5$; roots $= (3\sqrt{5} \pm \sqrt{5})/2 = 2\sqrt{5}$ and $\sqrt{5}$.

(vii) Multiply by 2: $x^2 - 2\sqrt{11}x + 2 = 0$; $D = 44 - 8 = 36$; roots $= (2\sqrt{11} \pm 6)/2 = \sqrt{11} + 3$ and $\sqrt{11} - 3$.

Q2. Find the roots by factorisation: (i) $2x^2+(5/3)x-2=0$ (ii) $(2/5)x^2-x-3/5=0$ (iii) $3\sqrt{2}x^2-5x-\sqrt{2}=0$ (iv) $3x^2+5\sqrt{5}x-10=0$ (v) $21x^2-2x+1/21=0$.

Solution

(i) Multiply by 3: $6x^2 + 5x - 6 = 0 = (3x - 2)(2x + 3)$; roots $2/3$ and $-3/2$.

(ii) Multiply by 5: $2x^2 - 5x - 3 = 0 = (2x + 1)(x - 3)$; roots $-1/2$ and 3.

(iii) $3\sqrt{2}x^2 - 5x - \sqrt{2} = (3\sqrt{2}x + \ldots)$ factors to give roots $\sqrt{2}$ and $-\sqrt{2}/6$.

(iv) $3x^2 + 5\sqrt{5}x - 10$ factors to give roots $\sqrt{5}/3$ and $-2\sqrt{5}$.

(v) Multiply by 21: $441x^2 - 42x + 1 = (21x - 1)^2 = 0$; equal roots, both $1/21$.

Part E — Long Answer Questions

Worked Samples

QS1. Check whether $6x^2 - 7x + 2 = 0$ has real roots; if so, find them by completing the square.

Solution

Discriminant = $49 - 4(6)(2) = 49 - 48 = 1 > 0$, so two distinct real roots exist. Multiply the equation by 6 to make the leading term a perfect square: $36x^2 - 42x + 12 = 0$, i.e. $(6x - 7/2)^2 = (1/2)^2$. Taking square roots: $6x - 7/2 = \pm 1/2$, so $6x = 4$ or 3, giving $x = 2/3$ and $1/2$.

Answer: **Roots: 2/3 and 1/2.**

QS2. Had Ajita scored 10 more marks (out of 30), nine times that would equal the square of her actual marks. Find her marks.

Solution

Let her actual marks be x. Then $9(x + 10) = x^2 \rightarrow x^2 - 9x - 90 = 0 \rightarrow (x - 15)(x + 6) = 0 \rightarrow x = 15$ or $x = -6$. Marks cannot be negative, so $x = 15$.

Answer: **Ajita scored 15 marks.**

QS3. A train covers 63 km at a certain speed, then 72 km at 6 km/h more, taking 3 hours total. Find the original speed.

Solution

Let the original speed be x km/h. Time = distance ÷ speed: $63/x + 72/(x+6) = 3$. Clearing denominators leads to $x^2 - 39x - 126 = 0 \rightarrow (x - 42)(x + 3) = 0 \rightarrow x = 42$ or $x = -3$. Speed cannot be negative, so $x = 42$.

Answer: **Original average speed = 42 km/h.**

Exercise 4.4

Q1. Find whether real roots exist; if so, find them: (i) $8x^2+2x-3=0$ (ii) $-2x^2+3x+2=0$ (iii) $5x^2-2x-10=0$ (iv) $1/(2x-3) + 1/(x-5) = 1$ (v) $x^2+5\sqrt{5}x-70=0$.

Solution

(i) $D = 4 + 96 = 100 > 0$; roots $= (-2 \pm 10)/16 = 1/2$ and $-3/4$.

(ii) Multiply by -1: $2x^2 - 3x - 2 = 0$; $D = 9 + 16 = 25 > 0$; roots $= (3 \pm 5)/4 = 2$ and $-1/2$.

(iii) $D = 4 + 200 = 204 > 0$; roots $= (2 \pm \sqrt{204})/10 = (1 \pm \sqrt{51})/5$.

(iv) Combine the fractions: $(x - 5) + (2x - 3) = (2x - 3)(x - 5) \rightarrow 3x - 8 = 2x^2 - 13x + 15 \rightarrow 2x^2 - 16x + 23 = 0$; $D = 256 - 184 = 72 > 0$; roots $= (16 \pm 6\sqrt{2})/4 = 4 \pm (3\sqrt{2})/2$.

(v) $D = 125 + 280 = 405 > 0$; roots $= (-5\sqrt{5} \pm \sqrt{405})/2 = (-5\sqrt{5} \pm 9\sqrt{5})/2 = 2\sqrt{5}$ and $-7\sqrt{5}$.

Q2. Find a natural number whose square diminished by 84 equals thrice the number more than 8.

Solution

Let the number be n. Then $n^2 - 84 = 3(n + 8) \rightarrow n^2 - 3n - 108 = 0 \rightarrow (n - 12)(n + 9) = 0 \rightarrow n = 12$ or $n = -9$. A natural number must be positive, so $n = 12$.

Answer: The natural number is 12.

Q3. A natural number increased by 12 equals 160 times its reciprocal. Find it.

Solution

Let the number be n. Then $n + 12 = 160/n \rightarrow n^2 + 12n - 160 = 0 \rightarrow (n - 8)(n + 20) = 0 \rightarrow n = 8$ or $n = -20$. Taking the natural number, $n = 8$.

Answer: The natural number is 8.

Q4. A train covering 360 km would take 48 minutes less if its speed were 5 km/h more. Find the original speed.

Solution

Let the speed be x km/h. The time saved is 48 minutes = 48/60 = 4/5 hour: $360/x - 360/(x+5) = 4/5$. Clearing denominators gives $x^2 + 5x - 2250 = 0 \rightarrow (x - 45)(x + 50) = 0 \rightarrow x = 45$ or $x = -50$. Speed is positive, so $x = 45$.

Answer: Original speed = 45 km/h.

Q5. If Zeba were 5 years younger, the square of her age would have been 11 more than five times her actual age. Find her age.

Solution

Let her actual age be a. Then $(a - 5)^2 = 5a + 11 \rightarrow a^2 - 10a + 25 = 5a + 11 \rightarrow a^2 - 15a + 14 = 0 \rightarrow (a - 14)(a - 1) = 0 \rightarrow a = 14$ or $a = 1$. Age 1 makes 'younger by 5' impossible, so $a = 14$.

Answer: Zeba is 14 years old now.

Q6. Asha's age is 2 more than the square of Nisha's age. When Nisha reaches Asha's present age, Asha will be one year less than ten times Nisha's present age. Find both ages.

Solution

Let Nisha's age be N. Then Asha = $N^2 + 2$. The gap between them stays constant: when Nisha is $N^2 + 2$, Asha will be $(N^2 + 2) + (N^2 + 2 - N)$. Setting this equal to $10N - 1$: $2N^2 - N + 4 = 10N - 1 \rightarrow 2N^2 - 11N + 5 = 0 \rightarrow (2N - 1)(N - 5) = 0 \rightarrow N = 5$ (rejecting the fractional value). Then Asha = 25 + 2 = 27.

Answer: Nisha is 5 years old, Asha is 27 years old.

Q7. A 50 m × 40 m lawn has a central rectangular pond, leaving 1184 m^2 of grass around it. Find the pond's length and breadth.

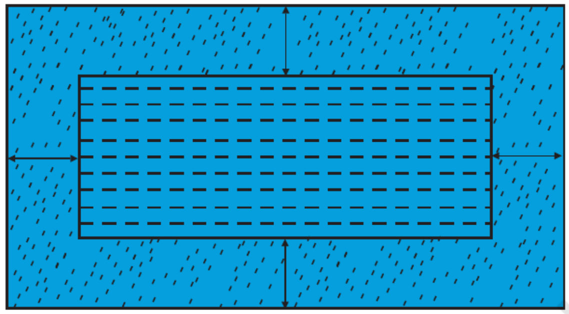

Fig. 4.1

Solution

Let the uniform grass margin be x metres wide, so the pond measures $(50 - 2x)$ by $(40 - 2x)$. Lawn area = 2000 m^2, grass = 1184 m^2, so pond area = $2000 - 1184 = 816$. Then $(50 - 2x)(40 - 2x) = 816 \rightarrow 4x^2 - 180x + 2000 = 816 \rightarrow x^2 - 45x + 296 = 0 \rightarrow (x - 8)(x - 37) = 0 \rightarrow x = 8$ ($x = 37$ is too large). So the pond is $(50 - 16)$ by $(40 - 16)$ = 34 m by 24 m.

Answer: Pond length 34 m, breadth 24 m.

Q8. At t minutes past 2 pm, the time the minute hand still needs to reach 3 pm is 3 minutes less than $t^2/4$ minutes. Find t.

Solution

From t minutes past 2 pm, the time left to reach 3 pm is $(60 - t)$ minutes. The condition says $60 - t = t^2/4 - 3$. Multiplying by 4: $240 - 4t = t^2 - 12 \rightarrow t^2 + 4t - 252 = 0 \rightarrow (t - 14)(t + 18) = 0 \rightarrow t = 14$ (rejecting the negative value).

Answer: $t = 14$ minutes.

Quick Revision Sheet (Night Before the Exam)

- Standard form: $ax^2 + bx + c = 0$ with $a \neq 0$. Roots = zeroes of the polynomial.
- Discriminant $D = b^2 - 4ac$: $D > 0 \rightarrow$ two distinct roots; $D = 0 \rightarrow$ equal roots; $D < 0 \rightarrow$ no real roots.
- Quadratic formula: $x = [-b \pm \sqrt{(b^2 - 4ac)}] / 2a$.
- Sum of roots = $-b/a$; product of roots = c/a — great for quick checks.
- Factorisation: split the middle term into two numbers that multiply to $a \cdot c$ and add to b.
- Completing the square: scale so the leading coefficient is a perfect square, form $(\ldots\ldots)^2 = (\ldots\ldots)^2$, then take roots.
- Word-problem setups: speed–distance–time (time = distance/speed, build the difference equation), ages (shift everyone by the same gap), numbers (translate 'reciprocal', 'square', 'diminished by' literally), geometry (area = length × breadth, use a margin variable for borders).
- Always reject the root that breaks physical sense (negative age, speed, marks, or count).