

CLASS 10 MATHEMATICS | NCERT EXEMPLAR | CHAPTER 8

TRIGONOMETRY

Ratios, Identities & Applications — Complete Study Guide with Diagrams

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STANDARD VALUES TABLE — MEMORISE

DIAGRAM — Trig Values Table

Angle	0 deg	30 deg	45 deg	60 deg	90 deg
	0	1/2	1/sqrt(2)	sqrt(3)/2	1
cos A	1	sqrt(3)/2	1/sqrt(2)	1/2	0
	0	1/sqrt(3)	1	sqrt(3)	N/D
cosec A	N/D	2	sqrt(2)	2/sqrt(3)	1
	1	2/sqrt(3)	sqrt(2)	2	N/D
cot A	N/D	sqrt(3)	1	1/sqrt(3)	0

TIP: Memory trick for sin: sin 0=0, sin 30=1/2, sin 45=1/sqrt(2), sin 60=sqrt(3)/2, sin 90=1. Pattern: sqrt(0)/2, sqrt(1)/2, sqrt(2)/2, sqrt(3)/2, sqrt(4)/2. Cos is exactly reversed!

KEY IDENTITIES & COMPLEMENTARY ANGLES

Identity 1	$\sin^2 A + \cos^2 A = 1$ (most used)
Identity 2	$1 + \tan^2 A = \sec^2 A$
Identity 3	$1 + \cot^2 A = \operatorname{cosec}^2 A$
Complementary	$\sin(90-A)=\cos A, \cos(90-A)=\sin A, \tan(90-A)=\cot A$
Complementary	$\sec(90-A)=\operatorname{cosec} A, \operatorname{cosec}(90-A)=\sec A, \cot(90-A)=\tan A$

tan & cot

$$\tan A = \sin A / \cos A; \cot A = \cos A / \sin A$$

MCQ EXERCISE 8.1 — Multiple Choice Questions**Q1** If $\cos A = 4/5$, find $\tan A$.(A) $3/5$ **(B) [OK] $3/4$** (C) $4/3$ (D) $5/3$ **Solution — Q1**
 $\cos A = 4/5 \rightarrow \text{adj} = 4, \text{hyp} = 5 \rightarrow \text{opp} = \sqrt{25 - 16} = 3. \tan A = \text{opp}/\text{adj} = 3/4.$
[OK] $\tan A = 3/4$ **Q2** If $\sin A = 1/2$, find $\cot A$.**(A) [OK] $\sqrt{3}$** (B) $1/\sqrt{3}$ (C) $\sqrt{3}/2$

(D) 1

Solution — Q2
 $\sin A = 1/2 \rightarrow A = 30^\circ. \cot 30 = \cos 30 / \sin 30 = (\sqrt{3}/2) / (1/2) = \sqrt{3}.$
[OK] $\cot A = \sqrt{3}$ **Q3** Value of $[\operatorname{cosec}(75+t) - \sec(15-t) - \tan(55+t) + \cot(35-t)]$:

(A) -1

(B) [OK] 0

(C) 1

(D) $3/2$ **Solution — Q3**
 $\operatorname{cosec}(75+t) = \sec(15-t)$ [complementary]. First two cancel.

 $\cot(35-t) = \tan(55+t)$ [complementary]. Last two cancel.
[OK] 0**Q4** If $\sin(t) = a/b$, find $\cos(t)$:(A) $b/\sqrt{b^2 - a^2}$ (B) b/a **(C) [OK] $\sqrt{b^2 - a^2}/b$** (D) $a/\sqrt{b^2 - a^2}$ **Solution — Q4**
 $\text{opp} = a, \text{hyp} = b \rightarrow \text{adj} = \sqrt{b^2 - a^2}. \cos(t) = \text{adj}/\text{hyp}.$
[OK] $\cos(t) = \sqrt{b^2 - a^2}/b$

Q5 If $\cos(a+b)=0$, then $\sin(a-b)$ can be reduced to:

(A) $\cos b$

(B) [OK] $\cos 2b$

(C) $\sin a$

(D) $\sin 2a$

Solution — Q5

$\cos(a+b)=0 \rightarrow a+b=90^\circ \rightarrow b=90-a$.

$\sin(a-b)=\sin(2a-90)=-\cos 2a$. Also $\cos 2b=\cos(180-2a)=-\cos 2a$.

[OK] $\cos 2b$

Q6 Value of $\tan 1^\circ \times \tan 2^\circ \times \tan 3^\circ \times \dots \times \tan 89^\circ$:

(A) 0

(B) [OK] 1

(C) 2

(D) $1/2$

Solution — Q6

Pair: $\tan k^\circ \times \tan(90-k)^\circ = \tan k^\circ \times \cot k^\circ = 1$. $\tan 45^\circ = 1$. Product = 1.

[OK] 1

Q7 If $\cos 9a = \sin(a)$ and $9a < 90^\circ$, find $\tan 5a$:

(A) $1/\sqrt{3}$

(B) $\sqrt{3}$

(C) [OK] 1

(D) 0

Solution — Q7

$\cos 9a = \cos(90-a) \rightarrow 9a = 90-a \rightarrow a = 9^\circ$. $\tan 5a = \tan 45^\circ = 1$.

[OK] 1

Q8 Triangle ABC right-angled at C. Find $\cos(A+B)$:

(A) [OK] 0

(B) 1

(C) $1/2$

(D) $\sqrt{3}/2$

Solution — Q8

$A+B+C=180$. $C=90 \rightarrow A+B=90^\circ$. $\cos 90^\circ = 0$.

[OK] 0

Q9 If $\sin A + \sin^2 A = 1$, find $\cos^2 A + \cos^4 A$:

(A) [OK] 1

(B) $\frac{1}{2}$

(C) 2

(D) 3

Solution — Q9

$$\sin A + \sin^2 A = 1 \Rightarrow \cos^2 A = 1 - \sin^2 A = \cos^2 A$$

$$\cos^2 A + \cos^4 A = \cos^2 A + (\cos^2 A)^2 = \sin A + \sin^2 A = 1.$$

[OK] 1

Q10 $\sin(a) = \frac{1}{2}$ and $\cos(b) = \frac{1}{2}$. Find $(a+b)$:

(A) 0 deg

(B) 30 deg

(C) 60 deg

(D) [OK] 90 deg

Solution — Q10

$$\sin(a) = \frac{1}{2} \Rightarrow a = 30^\circ, \cos(b) = \frac{1}{2} \Rightarrow b = 60^\circ, a+b = 90^\circ.$$

[OK] 90 deg

Q11 Value of $\frac{(\sin^2 22^\circ + \sin^2 68^\circ)}{(\cos^2 22^\circ + \cos^2 68^\circ)} + \sin^2 63^\circ + \cos 63^\circ \sin 27^\circ$:

(A) 3

(B) [OK] 2

(C) 1

(D) 0

Solution — Q11

$$\sin 68^\circ = \cos 22^\circ: \text{numerator} = \sin^2 22^\circ + \cos^2 22^\circ = 1, \text{denom} = 1. \text{Part1} = 1.$$

$$\sin 27^\circ = \cos 63^\circ: \sin^2 63^\circ + \cos^2 63^\circ = 1. \text{Part2} = 1.$$

[OK] $1+1=2$

Q12 If $4 \tan(t) = 3$, find $\frac{4\sin - \cos}{4\sin + \cos}$:

(A) $\frac{2}{3}$

(B) $\frac{1}{3}$

(C) [OK] $\frac{1}{2}$

(D) $\frac{3}{4}$

Solution — Q12

$$\text{Divide by } \cos: \frac{4\tan - 1}{4\tan + 1} = \frac{3 - 1}{3 + 1} = \frac{2}{4} = \frac{1}{2}.$$

[OK] $\frac{1}{2}$

Q13 If $\sin(t) - \cos(t) = 0$, find $\sin^4(t) + \cos^4(t)$:

(A) 1

(B) $\frac{3}{4}$

(C) [OK] $\frac{1}{2}$

(D) $\frac{1}{4}$

Solution — Q13

$\sin = \cos \rightarrow t = 45^\circ$. $\sin^2 = \cos^2 = \frac{1}{2}$.

$\sin^4 + \cos^4 = (\frac{1}{2})^2 + (\frac{1}{2})^2 = \frac{1}{2}$.

[OK] $\frac{1}{2}$

Q14 $\sin(45+t) - \cos(45-t)$ equals:

(A) $2\cos t$

(B) [OK] 0

(C) $2\sin t$

(D) 1

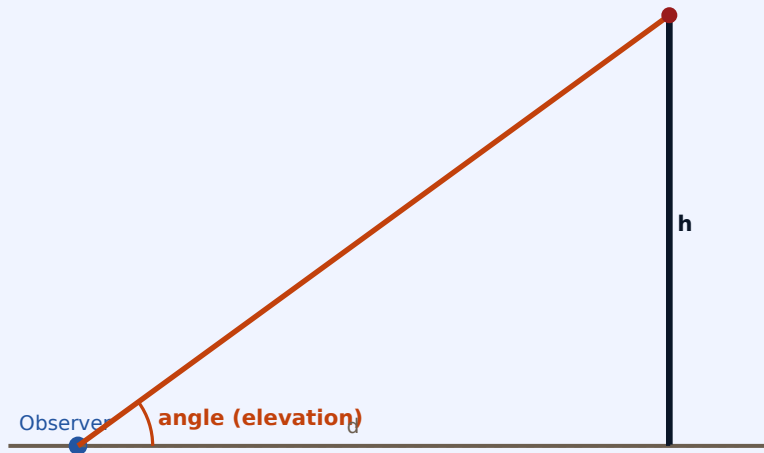
Solution — Q14

$\cos(45-t) = \sin(45+t)$ [complementary]. Result = $\sin(45+t) - \sin(45+t) = 0$.

[OK] 0

Q15 Pole 6m high, shadow $2\sqrt{3}$ m. Sun's elevation:

DIAGRAM — Q15



$$\tan(\text{elevation}) = h/d$$

(A) [OK] 60 deg

(B) 45 deg

(C) 30 deg

(D) 90 deg

Solution — Q15

$$\tan(t) = 6/(2\sqrt{3}) = 3/\sqrt{3} = \sqrt{3}. \quad \tan(t) = \sqrt{3} \rightarrow t = 60 \text{ deg.}$$

[OK] 60 deg

T/F EXERCISE 8.2 — True or False

Q1 $\tan 47^\circ / \cot 43^\circ = 1$?

ANSWER: TRUE

$$\cot 43^\circ = \cot(90^\circ - 47^\circ) = \tan 47^\circ. \text{ So } \tan 47^\circ / \tan 47^\circ = 1.$$

Q2 Value of $(\cos^2 23^\circ - \sin^2 67^\circ)$ is positive?

ANSWER: FALSE

$$\sin 67^\circ = \cos 23^\circ \rightarrow \cos^2 23^\circ - \sin^2 67^\circ = \cos^2 23^\circ - \cos^2 23^\circ = 0. \text{ NOT positive.}$$

Q3 Value of $(\sin 80^\circ - \cos 80^\circ)$ is negative?

ANSWER: FALSE

$$\text{For angles } > 45^\circ, \sin > \cos. \sin 80^\circ > \cos 80^\circ \rightarrow \text{difference is POSITIVE.}$$

Q4 $\sqrt{(1-\cos^2 t) \cdot \sec^2 t} = \tan t$?

ANSWER: TRUE

$1-\cos^2 t = \sin^2 t$. $\sqrt{(\sin^2 t / \cos^2 t)} = |\tan t| = \tan t$ for $0 < t < 90^\circ$.

Q5 If $\cos A + \cos^2 A = 1$, then $\sin^2 A + \sin^4 A = 1$?

ANSWER: TRUE

$\cos A = \sin^2 A$. $\sin^2 A + \sin^4 A = \sin^2 A + (\sin^2 A)^2 = \sin^2 A + \cos^2 A = 1$.

Q6 $(\tan t + 2)(2\tan t + 1) = 5\tan t + \sec^2 t$?

ANSWER: FALSE

LHS = $2\tan^2 t + 5\tan t + 2$. RHS = $\tan^2 t + 5\tan t + 1$. Not equal.

Q7 If shadow of tower is increasing, angle of elevation of sun is also increasing?

ANSWER: FALSE

Longer shadow means sun is lower $\rightarrow$ angle DECREASES.

Q8 Man 3m above lake observes cloud and reflection. Elevation = depression angle?

ANSWER: FALSE

Cloud at height H : elevation = $\arctan[(H-3)/d]$. Depression of reflection = $\arctan[(H+3)/d]$. Not equal.

Q9 $2 \sin(t)$ can equal $(a+1/a)$ for positive a not equal to 1?

ANSWER: FALSE

For $a > 0$, $a + 1/a \geq 2$ (AM-GM). When $a \neq 1$, $a + 1/a > 2$, so $2\sin(t) > 2 \rightarrow \sin > 1$. Impossible.

Q10 $\cos(t) = (a^2 + b^2)/(2ab)$ for distinct a, b with $ab > 0$?

ANSWER: FALSE

AM-GM: $a^2 + b^2 \geq 2ab$, strict when $a \neq b$. So $(a^2 + b^2)/(2ab) > 1$. But $\cos \leq 1$. Impossible.

Q11 Tower height doubles $\rightarrow$ angle of elevation doubles?

ANSWER: FALSE

$\tan(t)$ doubles but t does not double. $\arctan(2 \cdot \tan t)$ is not $2t$ in general.

Q12 Height and distance both increased by 10% $\rightarrow$ angle unchanged?

ANSWER: TRUE

$\tan(t) = h/d$. New $\tan = (1.1h)/(1.1d) = h/d$. Same ratio $\rightarrow$ same angle.

SA EXERCISE 8.3 — Short Answer (Proofs & Problems)

Q1 Prove: $\sin t/(1+\cos t) + (1+\cos t)/\sin t = 2 \operatorname{cosec} t$

Proof — Q1

$$\begin{aligned} \text{LHS} &= [\sin^2 t + (1+\cos t)^2] / [\sin t(1+\cos t)] \\ &= [\sin^2 t + 1 + 2\cos t + \cos^2 t] / [\sin t(1+\cos t)] \\ &= [2 + 2\cos t] / [\sin t(1+\cos t)] \\ &= 2(1+\cos t) / [\sin t(1+\cos t)] = 2/\sin t = 2 \operatorname{cosec} t. \\ &\Rightarrow \text{Proved} \end{aligned}$$

Q3 If $\tan A = 3/4$, prove $\sin A * \cos A = 12/25$.

Proof — Q3

$$\begin{aligned} \tan A &= 3/4 \rightarrow \text{opp}=3, \text{adj}=4, \text{hyp}=5. \\ \sin A &= 3/5, \cos A = 4/5. \\ \sin A * \cos A &= (3/5)*(4/5) = 12/25. \\ &\Rightarrow \text{Proved} \end{aligned}$$

Q4 Prove: $(\sin a + \cos a)(\tan a + \cot a) = \sec a + \operatorname{cosec} a$

Proof — Q4

$$\begin{aligned} \tan a + \cot a &= 1/(\sin a * \cos a). \\ \text{LHS} &= (\sin a + \cos a)/(\sin a * \cos a) = 1/\cos a + 1/\sin a = \sec a + \operatorname{cosec} a. \\ &\Rightarrow \text{Proved} \end{aligned}$$

Q6 Prove: $1 + \cot^2(a)/(1+\operatorname{cosec} a) = \operatorname{cosec} a$

Proof — Q6

$$\begin{aligned} \cot^2 a &= \operatorname{cosec}^2 a - 1 = (\operatorname{cosec} a + 1)(\operatorname{cosec} a - 1). \\ \text{LHS} &= [1 + \operatorname{cosec} a + (\operatorname{cosec} a + 1)(\operatorname{cosec} a - 1)] / (1 + \operatorname{cosec} a) \\ &= (1 + \operatorname{cosec} a)(\operatorname{cosec} a) / (1 + \operatorname{cosec} a) = \operatorname{cosec} a. \\ &\Rightarrow \text{Proved} \end{aligned}$$

Q7 Prove: $\tan t + \tan(90-t) = \sec t * \sec(90-t)$

Proof — Q7

$$\text{LHS} = \tan t + \cot t = 1/(\sin t * \cos t).$$

$$\text{RHS} = \sec t * \operatorname{cosec} t = 1/(\cos t * \sin t).$$

$$\Rightarrow \text{LHS} = \text{RHS} \text{ (Proved)}$$

Q8 Find angle of elevation when shadow of h-metre pole is $\sqrt{3}h$ metres.

Solution — Q8

$$\tan(t) = h/(\sqrt{3}h) = 1/\sqrt{3}.$$

$$\text{[OK] } t = 30 \text{ deg}$$

Q9 If $\sqrt{3} \tan(t) = 1$, find $\sin^2 t - \cos^2 t$.

Solution — Q9

$$\tan(t) = 1/\sqrt{3} \rightarrow t = 30 \text{ deg.}$$

$$\sin^2 30 - \cos^2 30 = 1/4 - 3/4 = -1/2.$$

$$\text{[OK] } -1/2$$

Q10 A 15m ladder makes 60 deg angle with the wall. Find height reached.

Solution — Q10

$$\text{Angle with wall} = 60 \text{ deg. Height} = \text{ladder} * \cos 60 = 15 * (1/2) = 7.5 \text{ m.}$$

$$\text{[OK] Height} = 7.5 \text{ m}$$

Q11 Simplify $(1 + \tan^2 t)(1 - \sin t)(1 + \sin t)$.

Solution — Q11

$$(1 - \sin t)(1 + \sin t) = \cos^2 t.$$

$$(1 + \tan^2 t) = \sec^2 t.$$

$$\sec^2 t * \cos^2 t = 1.$$

$$\text{[OK] Result} = 1$$

Q12 If $2\sin^2 t - \cos^2 t = 2$, find t.

Solution — Q12

$$\text{Replace } \cos^2 = 1 - \sin^2: 3\sin^2 - 1 = 2 \rightarrow \sin^2 = 1 \rightarrow \sin = 1 \rightarrow t = 90 \text{ deg.}$$

$$\text{[OK] } t = 90 \text{ deg}$$

Q14 Observer 1.5m tall, 20.5m from tower 22m high. Angle of elevation from observer's eye?

Solution — Q14

Effective height = $22 - 1.5 = 20.5$ m. Horizontal = 20.5 m.

$\tan(t) = 20.5/20.5 = 1 \rightarrow t = 45^\circ$.

[OK] 45 deg

LA EXERCISE 8.4 — Long Answer Questions

Q1 If $\operatorname{cosec} t + \cot t = p$, prove $\cos t = (p^2 - 1)/(p^2 + 1)$.

Solution — Q1

$p = \operatorname{cosec} t + \cot t = (1 + \cos t)/\sin t$.

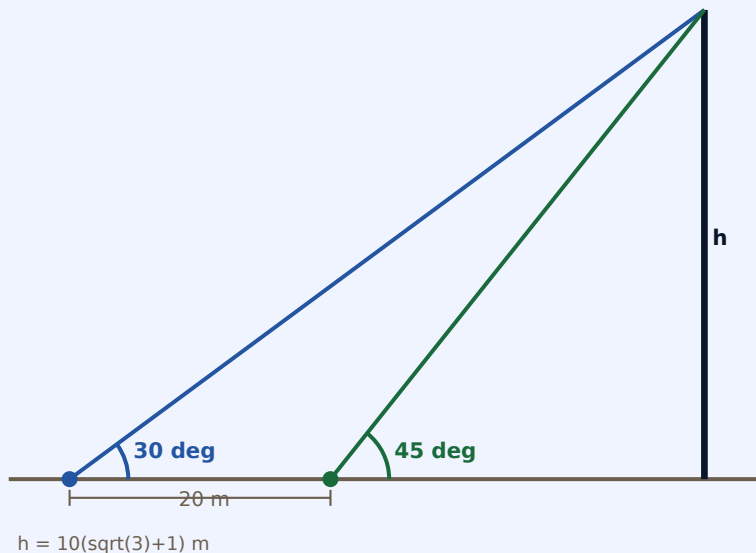
$p^2 = (1 + \cos t)^2/\sin^2 t = (1 + \cos t)/(1 - \cos t)$.

$p^2(1 - \cos t) = 1 + \cos t \Rightarrow \cos t(1 + p^2) = p^2 - 1$.

$\Rightarrow \cos t = (p^2 - 1)/(p^2 + 1)$ (Proved)

Q3 Elevation is 30° . Observer moves 20m closer; elevation becomes 45° . Find height.

DIAGRAM — Q3



Solution — Q3

Let height = h , original dist = d . $\tan 30 = h/d \rightarrow d = h\sqrt{3}$.

New dist = $d - 20$. $\tan 45 = h/(d - 20) \rightarrow d - 20 = h$.

$h\sqrt{3} - 20 = h \rightarrow h(\sqrt{3} - 1) = 20 \rightarrow h = 10(\sqrt{3} + 1) \text{ m}$.

$\Rightarrow \text{Height} = 10(\sqrt{3} + 1) \text{ m}$

Q6 Elevation from two points distance s and t are complementary. Prove $\text{height} = \sqrt{st}$.

Solution — Q6

Angles: a and $(90-a)$. $\tan a = h/s \rightarrow h = s \tan a \dots (1)$

$\tan(90-a) = h/t \rightarrow \cot a = h/t \rightarrow h = t \cot a \dots (2)$

Multiply: $h^2 = s \tan a \cdot t \cot a = st$.

$\Rightarrow h = \sqrt{st}$ (Proved)

Q7 Shadow is 50m longer when elevation = 30 deg than when 60 deg. Find tower height.

Solution — Q7

$d_1 = h/\tan 60 = h/\sqrt{3}$. $d_2 = h/\tan 30 = h\sqrt{3}$.

$d_2 - d_1 = h\sqrt{3} - h/\sqrt{3} = h(2/\sqrt{3}) = 50 \rightarrow h = 25\sqrt{3} \text{ m}$.

$\Rightarrow \text{Height} = 25\sqrt{3} \text{ m}$

Q9 If $\tan t + \sec t = l$, prove $\sec t = (l^2 + 1)/(2l)$.

Solution — Q9

$\tan t + \sec t = l \dots (1)$

$\sec^2 t - \tan^2 t = 1 \rightarrow (\sec + \tan)(\sec - \tan) = 1 \rightarrow \sec t - \tan t = 1/l \dots (2)$

Adding (1)+(2): $2\sec t = l + 1/l = (l^2 + 1)/l$.

$\Rightarrow \sec t = (l^2 + 1)/(2l)$ (Proved)

Q13 Tower A is 30m high. Elevation of A from foot of B = 60 deg. Elevation of B from foot of A = 30 deg. Find distance and height of B.

Solution — Q13

From foot of B: $\tan 60 = 30/d \rightarrow d = 30/\sqrt{3} = 10\sqrt{3} \text{ m}$.

From foot of A: $\tan 30 = h/d \rightarrow h = 10\sqrt{3} \cdot (1/\sqrt{3}) = 10 \text{ m}$.

$\Rightarrow \text{Distance} = 10\sqrt{3} \text{ m}$, $\text{Height of tower B} = 10 \text{ m}$

Q16 Elevation from ground = 60 deg. From 10m above, elevation = 45 deg. Find height.

Solution — Q16

Height = H , distance = d . $\tan 60 = H/d \rightarrow d = H/\sqrt{3} \dots (1)$

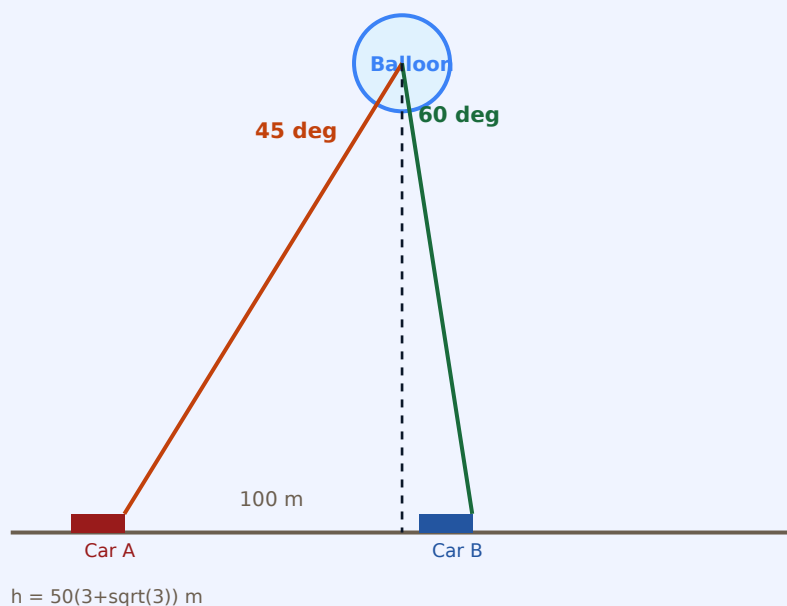
$\tan 45 = (H-10)/d \rightarrow d = H-10 \dots (2)$

$H/\sqrt{3} = H-10 \rightarrow H(\sqrt{3}-1) = 10\sqrt{3} \rightarrow H = 5(3+\sqrt{3}) = (15+5\sqrt{3}) \text{ m}$.

$\Rightarrow \text{Height} = (15 + 5\sqrt{3}) \text{ m}$

- Q18** Lower window 2m above ground, upper window 6m above. Elevation from lower=60, upper=30. Find balloon height.

DIAGRAM — Q18

**Solution — Q18**

Let balloon height = H above ground. Horizontal distance = d .

From lower window: $\tan 60 = \frac{H-2}{d} \rightarrow d = \frac{H-2}{\sqrt{3}}$... (1)

From upper window: $\tan 30 = \frac{H-6}{d} \rightarrow d = (H-6)\sqrt{3}$... (2)

$\frac{H-2}{\sqrt{3}} = (H-6)\sqrt{3} \rightarrow H-2 = 3(H-6) \rightarrow 2H = 16.$

$\Rightarrow H = 8$ m (Balloon is 8 m above ground)

REVISION QUICK REVISION — ANSWER KEY

Ex 8.1 MCQ	Q1- $\rightarrow$ B, Q2- $\rightarrow$ A, Q3- $\rightarrow$ B, Q4- $\rightarrow$ C, Q5- $\rightarrow$ B, Q6- $\rightarrow$ B, Q7- $\rightarrow$ C, Q8- $\rightarrow$ A
Ex 8.1 MCQ	Q9- $\rightarrow$ A, Q10- $\rightarrow$ D, Q11- $\rightarrow$ B, Q12- $\rightarrow$ C, Q13- $\rightarrow$ C, Q14- $\rightarrow$ B, Q15- $\rightarrow$ A
Ex 8.2 T/F	Q1:T, Q2:F, Q3:F, Q4:T, Q5:T, Q6:F, Q7:F, Q8:F, Q9:F, Q10:F, Q11:F, Q12:T
Ex 8.3	Q8:30 deg Q9:-1/2 Q10:7.5m Q11:1 Q12:90 deg Q14:45 deg
Ex 8.4	Q3:10($\sqrt{3}+1$)m Q6: $\sqrt{st}$ Q7:25 $\sqrt{3}$ m Q9:(l^2+1)/(2l)
Ex 8.4	Q13:10 $\sqrt{3}$ m dist, 10m height Q16:(15+5 $\sqrt{3}$)m Q18:8m

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