MATHEMATICS

Code No. 041

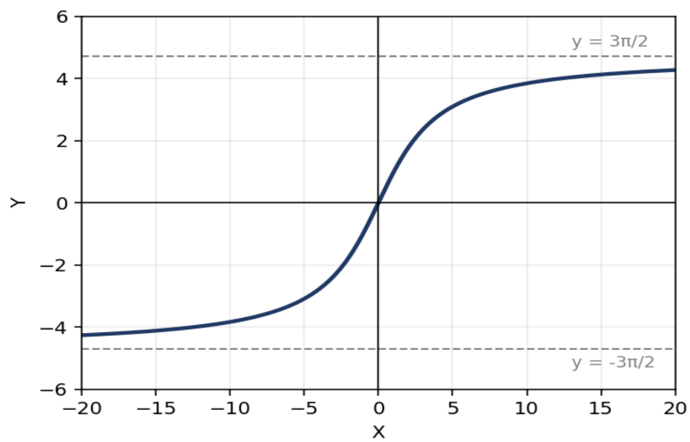
SAMPLE QUESTION PAPER — SET 3 | CLASS XII

Time Allowed: 3 Hours **Maximum Marks: 80**

General Instructions:

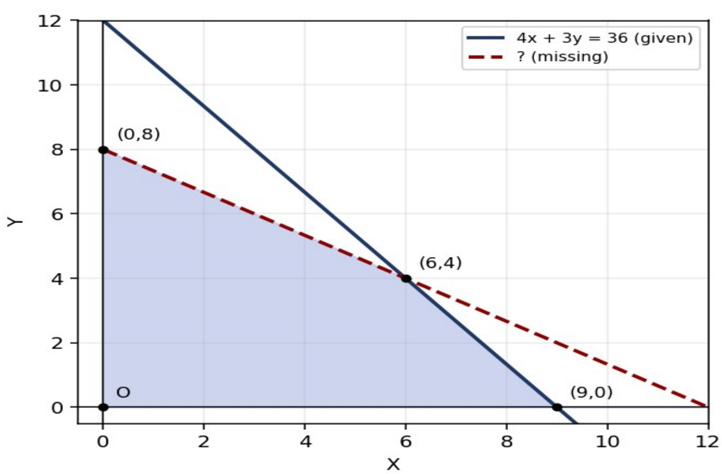
1. This question paper contains 38 questions. All questions are compulsory.
2. This question paper is divided into five Sections: A, B, C, D and E.
3. In Section A, Question numbers 1 to 18 are multiple choice questions (MCQs) with only one correct option, and Question numbers 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In Section B, Question numbers 21 to 25 are Very Short Answer (VSA) type questions, carrying 2 marks each.
5. In Section C, Question numbers 26 to 31 are Short Answer (SA) type questions, carrying 3 marks each.
6. In Section D, Question numbers 32 to 35 are Long Answer (LA) type questions, carrying 5 marks each.
7. In Section E, Question numbers 36 to 38 are Case-study based questions, carrying 4 marks each.
8. There is no overall choice. However, an internal choice has been provided in some questions in each of Sections B, C and D, and in one subpart of two questions in Section E.
9. Use of calculator is not allowed.

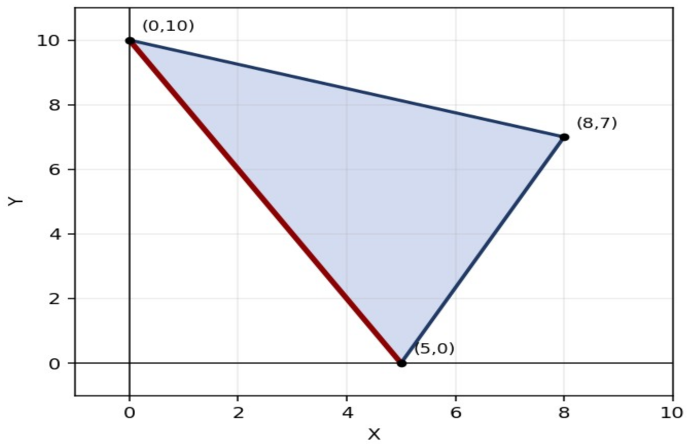
<table>
<tr><td colspan="3">SECTION A
This section comprises multiple choice questions (MCQs) of 1 mark each (Q1-18), and Assertion-Reason questions (Q19-20).</td></tr>
<tr><td>1.</td><td>Identify the function shown in the graph:



(A) $\tan^{-1}x$ (B) $\tan^{-1}(x/3)$ (C) $3\tan^{-1}(x/3)$ (D) $\tan^{-1}(3x)$</td><td>1</td></tr>
<tr><td>2.</td><td>If for three matrices $A=[a_{ij}]_{m\times 7}$, $B=[b_{ij}]_{n\times 4}$ and $D=[d_{ij}]_{u\times v}$, the products AB and AD are both defined and are square matrices of the same order, then the values of m,n,u,v are:
(A) m=v=4 and n=u=7
(B) m=7, v=4 and n=u=4
(C) m=u=7 and n=v=4
(D) m=4, u=7 and n=v=7</td><td>1</td></tr>
<tr><td>3.</td><td>If the matrix $A = [[0,m,-10],[8,n,p],[q,-7,0]]$ is skew-symmetric, then the value of $(q+p)/(n+m)$ is:</td><td>1</td></tr>
</table>

	(A) -17/8 (B) 17/8 (C) -8/17 (D) 8/17	
4.	If A is a square matrix of order 5 and |adj A| = 16, then A(adj A) is equal to: (A) 2 (B) 16 (C) 2I (D) 16I	1
5.	The inverse of the matrix [[6,0,0],[0,10,0],[0,0,4]] is: (A) [[1/6,0,0],[0,1/10,0],[0,0,1/4]] (B) [[6,0,0],[0,10,0],[0,0,4]] (C) [[-1/6,0,0],[0,-1/10,0],[0,0,-1/4]] (D) [[4,0,0],[0,10,0],[0,0,6]]	1
6.	The value of the determinant | cos63° sin63° ; sin27° cos27° | is: (A) 0 (B) 1/2 (C) √3/2 (D) 1	1
7.	If the function f(x) = {kx+7, x ≤ 4; 5x-9, x > 4} is continuous at x=4, the value of k is: (A) 1 (B) 2 (C) 3 (D) 4	1
8.	If f(x) = 2x $\tan^{-1}x$, then f'(1) is equal to: (A) π/2 + 1 (B) π/2 - 1 (C) π/4 + 1 (D) π/4 - 1	1
9.	The function f(x) = 20 - 4x - $5x^2$ is increasing on the interval: (A) (-∞, -2/5] (B) (-∞, 2/5) (C) [-2/5, ∞) (D) [-2/5, 2/5]	1
10.	The differential equation 3x dx + 2y dy = 0 represents a family of: (A) circles (B) ellipses (C) parabolas (D) hyperbolas	1
11.	If f(-x) = -f(x) for all x (i.e., f is an odd function), then the value of $\int_{-a}$^a f(x) dx is: (A) 0 (B) 2$\int_0$^a f(x) dx	1

	(C) a (D) 2a	
12.	If $\int x^{11} \sin^2(x^{12}) \cos(x^{12})\, dx = a \sin^3(x^{12}) + C$, then a is equal to: (A) -1/36 (B) 1/36 (C) 1/12 (D) 1/3	1
13.	A car travels in a straight line given by the vector $2\hat{i}+3\hat{j}-\hat{k}$. An observer stands beside a straight track given by $\vec{r}=(1+6\lambda)\hat{i}+2\lambda\hat{j}+3\lambda\hat{k}$. The projected length of the car's path on the track is: (A) 15/7 units (B) 7/15 units (C) 15/13 units (D) 13/15 units	1
14.	The distance of the point with position vector $8\hat{i}+5\hat{j}+6\hat{k}$ from the y-axis is: (A) 10 units (B) $\sqrt{61}$ units (C) 8 units (D) 6 units	1
15.	If $\vec{a}=5\hat{i}+4\hat{j}-3\hat{k}$, $\vec{b}=7\hat{i}-6\hat{j}+2\hat{k}$ and $\vec{c}=10\hat{i}+6\hat{j}-8\hat{k}$ are three given vectors, then $(2\vec{a}\cdot\hat{i})\hat{i} - (\vec{b}\cdot\hat{j})\hat{j} + (\vec{c}\cdot\hat{k})\hat{k}$ is the same as the vector: (A) $\vec{a}$ (B) $\vec{b}$ (C) $\vec{a}-\vec{b}$ (D) $\vec{c}$	1
16.	A student comes across an incomplete question: Maximise Z = 6x + 5y + 1, subject to x ≥ 0, y ≥ 0, 4x + 3y ≤ 36, ... (one constraint missing). The graph below is given for this LPP.  The missing constraint is: (A) 2x + 3y ≤ 24 (B) 3x + 2y ≥ 24 (C) 2x + 3y ≥ 24 (D) 3x + 2y ≤ 24	1
17.	The feasible region of an LPP is bounded, and the objective function Z = 2x + y attains its minimum value at more than one point. One of these points is (5, 0), as shown below.	1

<table>
<tr><td></td><td>


Then one of the other points at which Z attains its minimum value is:

(A) (4, 2) (B) (1, 9) (C) (6, 3) (D) (0, 0)
</td><td></td></tr>
<tr><td>18.</td><td>
A person observed the first 2 digits of a friend's 5-digit locker code. What is the probability of correctly guessing the remaining 3 digits in a single attempt?

(A) 1/1000

(B) 1/100

(C) 1/10

(D) 1
</td><td>1</td></tr>
<tr><td>19.</td><td>
Q19 and Q20 are Assertion (A) and Reason (R) based questions. Mark the correct choice as:

(A) Both (A) and (R) are true and (R) is the correct explanation of (A).

(B) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(C) (A) is true but (R) is false.

(D) (A) is false but (R) is true.

Assertion (A): Value of $\tan^{-1}(1) + \tan^{-1}(2) + \tan^{-1}(3)$ is π.

Reason (R): $\tan^{-1}x + \tan^{-1}y = \pi + \tan^{-1}[(x+y)/(1-xy)]$ when $xy > 1$.
</td><td>1</td></tr>
<tr><td>20.</td><td>
Assertion (A): If $\vec{a}$, $\vec{b}$, $\vec{c}$ are the position vectors of the vertices of a triangle, and $\vec{a}+\vec{b}+\vec{c}=\vec{0}$, then the origin is the centroid of the triangle.

Reason (R): The centroid of a triangle with vertices having position vectors $\vec{a},\vec{b},\vec{c}$ is $(\vec{a}+\vec{b}+\vec{c})/3$.
</td><td>1</td></tr>
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SECTION B

This section comprises 5 Very Short Answer (VSA) type questions of 2 marks each.
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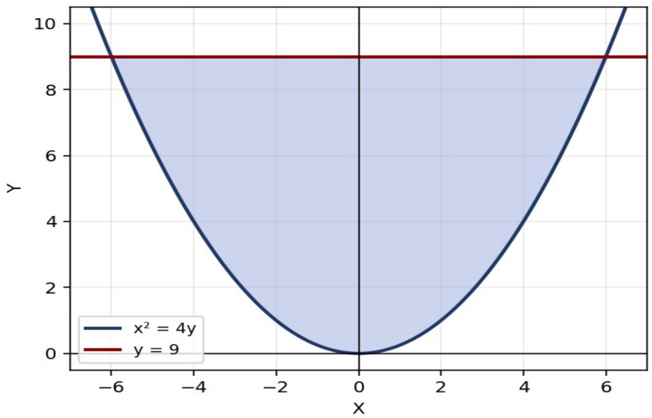
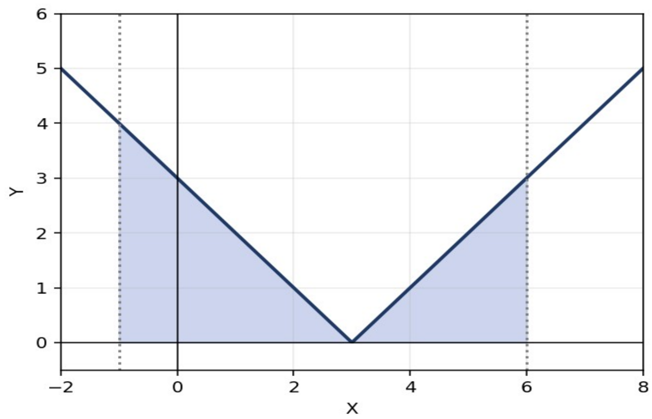
<tr><td>21.</td><td>
A. Evaluate: $\tan(\tan^{-1}(\sqrt{3}) - \pi/6)$

OR

B. Find the domain of $\sec^{-1}(2x-3)$.
</td><td>2</td></tr>
<tr><td>22.</td><td>
If $y = \log(\tan(x/2))$, prove that $dy/dx = \text{cosec}x$.
</td><td>2</td></tr>
</table>

<table>
<tr><td>23.</td><td>A. Find: $\int e^x (x-1)/x^2 \, dx$
OR
B. Find the area of the shaded region enclosed by the curve $x^2 = 4y$ and the line $y = 9$.

</td><td>2</td></tr>
<tr><td>24.</td><td>If $f(x+y) = f(x)f(y)$ for all $x, y \in R$, and $f(2) = 5$, $f'(0) = 4$, find $f'(2)$ using the definition of the derivative.</td><td>2</td></tr>
<tr><td>25.</td><td>The two vectors $3\hat{i}+\hat{j}+4\hat{k}$ and $5\hat{i}-3\hat{j}+2\hat{k}$ represent the sides OA and OB respectively of a $\triangle OAB$, where O is the origin. P is the midpoint of AB such that OP is a median of the triangle. Find the area of the parallelogram with adjacent sides OA and OP.</td><td>2</td></tr>
</table>

SECTION C *This section comprises 6 Short Answer (SA) type questions of 3 marks each.*		
26.	A. If $x^y = e^{(4(x-y))}$, find dy/dx in terms of x, and hence find its value at x = e. **OR** B. If $x = 2a(\theta + \sin\theta)$, $y = 2a(1 - \cos\theta)$, find d^2y/dx^2.	3
27.	A soap bubble is collapsing such that the rate of decrease of its volume at any instant is directly proportional to its surface area at that instant. Show that the radius of the bubble decreases at a constant rate.	3
28.	A. Sketch the graph $y = \|x-3\|$. Evaluate $\int_{-1}^{6} \|x-3\| \, dx$. What does this value represent on the graph?	3



OR

B. Using integration, find the area of the region enclosed by the parabola $y^2 = 16x$ and the line $x = 4$.

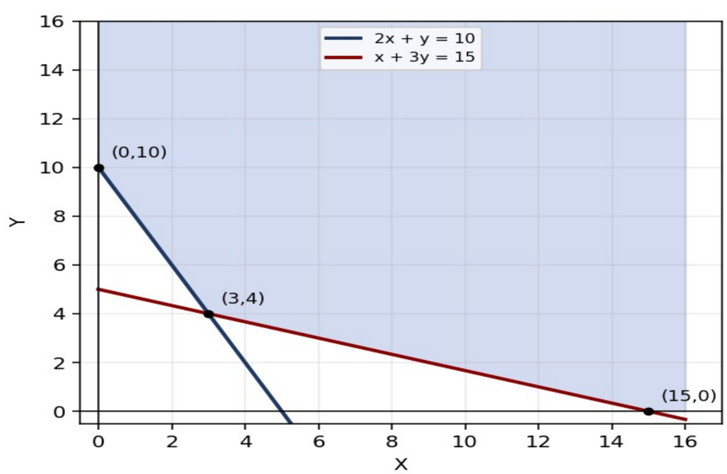
29. A. Find the distance of the point (6, 4, 11) from the line $\vec{r} = (6\hat{i}+4\hat{j}-3\hat{k}) + \mu(3\hat{i}+2\hat{j}+5\hat{k})$, measured parallel to the z-axis. **3**

OR

B. Find the point of intersection of the line $\vec{r} = (5\hat{i}+3\hat{k}) + \mu(2\hat{i}+\hat{j}-\hat{k})$ and the line through (3,-1,9) parallel to the z-axis. Also find the distance of this intersection point from the z-axis.

30. Solve the following linear programming problem graphically: **3**

Minimise $Z = 400x + 300y$, subject to the constraints: $2x + y \geq 10$, $x + 3y \geq 15$, $x \geq 0$, $y \geq 0$.



31. Two candidates, Aman and Bina, appear for an interview. The probability that Aman is selected is 0.7, and the probability that exactly one of them is selected is 0.4. Their selections are independent of each other. Find the probability that Bina is selected. Also find the probability that at least one of them is selected. **3**

SECTION D

This section comprises 4 Long Answer (LA) type questions of 5 marks each.

32.	For two matrices A = [[1,-1,2],[3,4,-5],[2,-1,3]] and B = [[7,1,-3],[-19,-1,11],[-11,-1,7]], find the product AB and hence solve the system of equations: $x - y + 2z = 9$; $3x + 4y - 5z = -13$; $2x - y + 3z = 14$	5
33.	A. Evaluate: $\int_0^{\pi/2} \sin x / (\sin x+\cos x)\, dx$ **OR** B. Find: $\int \cos x / (4-\sin^2 x)\, dx$	5
34.	A. Solve the differential equation: $d/dx(xy) = x(2\cos x - 3x)$ **OR** B. Find the particular solution of the differential equation: $(x^2+3y^2)dx - 2xy\, dy = 0$, given that $y(1) = 2$.	5
35.	The two lines $(x-2)/2 = (y-1)/1 = (z-9)/(-3)$ and $(x-3)/1 = (y-4)/(-2) = (z-2)/4$ intersect at a point whose y-coordinate is 2. Find the coordinates of their point of intersection. Also find the vector equation of the line perpendicular to both the given lines and passing through this point of intersection.	5

	SECTION E *This section comprises 3 case-study based questions of 4 marks each.*	
36.	**Case Study 1** An airline analyst is studying direct one-way flight connections between five airports M, N, O, P and Q. The following direct flights have been recorded: 1. Flights from M to N, and M to O. 2. A flight from N to P. 3. Flights from O to P, and O to Q. 4. A flight from P to Q. The analyst wants to represent and analyse this data as a relation on the set of airports. Use the given data to answer the following: (i) Is this relation reflexive? Justify. [1] (ii) Is this relation transitive? Justify. [1] (iii)(A) Represent the relation as a set of ordered pairs. Also state its domain and range. [2] **OR** (iii)(B) Does this relation represent a function from the set of airports to itself? Justify your answer. [2]	4
37.	**Case Study 2** A handicraft stall's cost of making x items, and the revenue generated from selling them, are modelled as: $C(x) = 0.6x^2 - 10x + 300$ and $R(x) = -0.4x^2 + 30x$, where C(x) and R(x) are both in ₹. To maximise profit, the stall owner needs to analyse these functions using calculus. Use the given models to answer the following:	4

	(i) Derive the profit function P(x). [1] (ii) Find the critical point of P(x). [1] (iii)(A) Determine whether the critical point corresponds to a maximum or minimum profit, using the second derivative test. Also find this profit. [2] **OR** (iii)(B) If the stall's resources allow it to make a minimum of 12 but not more than 28 items per hour, identify the practical value of x that maximises profit within this range, and calculate the maximum profit. [2]	
38.	**Case Study 3** Students are grouped by the time they spend reading before bed. Group 1 (more than 60 minutes) makes up 25% of students, Group 2 (20 to 60 minutes) makes up 45%, and Group 3 (less than 20 minutes) makes up 30%. It was found that 50% of Group 1, 20% of Group 2, and 10% of Group 3 students reported difficulty falling asleep. (i) What is the total percentage of students who report difficulty falling asleep? [2] (ii) A student is selected at random and is found to report difficulty falling asleep. What is the probability that this student belongs to Group 1 (more than 60 minutes of reading)? [2]	4

MATHEMATICS

Code No. 041 — Marking Scheme

MARKING SCHEME — SET 3 | CLASS XII

SECTION A		
1.	The curve is bounded with horizontal asymptotes at $y=3\pi/2$ and $y=-3\pi/2$, matching the range of $3\tan^{-1}(x/3)$ (since $\tan^{-1}$ itself is bounded by $\pm\pi/2$, scaled by 3). Answer: (C) $3\tan^{-1}(x/3)$	1
2.	Columns of A=rows of B: 7=n. AB is $m\times4$; square $\rightarrow$ m=4. Columns of A=rows of D: 7=u. AD is $m\times v=4\times v$; square (same order 4×4) $\rightarrow$ v=4. So m=v=4, n=u=7. Answer: (A)	1
3.	Skew-symmetric: diagonal=0 $\rightarrow$ n=0. $a_{12}=m, a_{21}=8 \rightarrow m=-8$. $a_{13}=-10, a_{31}=q \rightarrow q=10$. $a_{23}=p, a_{32}=-7 \rightarrow p=7$. $(q+p)/(n+m)=(10+7)/(0-8)=17/(-8)=-17/8$. Answer: (A) $-17/8$	1
4.	$\lvert adjA\rvert=\lvert A\rvert^{(n-1)}$; $n=5 \rightarrow \lvert A\rvert^4=16 \rightarrow \lvert A\rvert=2$. $A(adjA)=\lvert A\rvert I=2I$. Answer: (C) 2I	1
5.	Inverse of a diagonal matrix is the diagonal matrix of reciprocals. Answer: (A) [[1/6,0,0],[0,1/10,0],[0,0,1/4]]	1
6.	$\cos63°\cos27° - \sin63°\sin27° = \cos(63°+27°) = \cos90° = 0$. Answer: (A) 0	1
7.	At x=4: $4k+7 = 20-9=11 \rightarrow 4k=4 \rightarrow k=1$. Answer: (A) 1	1
8.	$f'(x)=2\tan^{-1}x + 2x/(1+x^2)$. $f'(1)=2(\pi/4)+2(1)/2=\pi/2+1$. Answer: (A) $\pi/2 + 1$	1
9.	$f'(x)=-4-10x\geq0 \rightarrow x\leq-2/5$. Answer: (A) $(-\infty,-2/5]$	1
10.	Integrating: $1.5x^2+y^2=C$, a family of ellipses. Answer: (B) ellipses	1
11.	For an odd function, the contributions from [-a,0] and [0,a] cancel exactly, so the integral over the symmetric interval is 0. Answer: (A) 0	1
12.	Let $u=x^{12}, du=12x^{11}dx$. Integral$=(1/12)(\sin^3u/3)+C=\sin^3u/36+C$. a=1/36. Answer: (B) 1/36	1
13.	Direction of track$=6\hat{i}+2\hat{j}+3\hat{k}$. $\vec{a}\cdot\vec{b}=2(6)+3(2)+(-1)(3)=12+6-3=15$. $\lvert\vec{b}\rvert=\sqrt{36+4+9}=\sqrt{49}=7$. Projection=15/7. Answer: (A) 15/7 units	1
14.	Distance from y-axis$=\sqrt{x^2+z^2}=\sqrt{64+36}=\sqrt{100}=10$. Answer: (A) 10 units	1
15.	$2(\vec{a}\cdot\hat{i})=2(5)=10 \rightarrow 10\hat{i}$. $-(\vec{b}\cdot\hat{j})=-(-6)=6 \rightarrow 6\hat{j}$. $(\vec{c}\cdot\hat{k})=-8 \rightarrow -8\hat{k}$. Sum$=10\hat{i}+6\hat{j}-8\hat{k}=\vec{c}$. Answer: (D) $\vec{c}$	1
16.	From the graph, the feasible region's boundary meets the x-axis at (9,0), consistent with $4x+3y=36$ binding there (since the alternative constraint would only bind at x=12); it meets the y-axis at (0,8), consistent with the missing line binding there. Solving $4x+3y=36$ with the missing line through (0,8) and (12,0)... checking $2x+3y=24$ gives exactly this. Answer confirmed by the intersection at (6,4). Answer: (A) $2x + 3y \leq 24$	1
17.	Since $Z=2x+y$ is minimised along the entire edge from (5,0) to (0,10), any point on this edge also	1

	gives the minimum value 10. Checking (4,2): $Z=2(4)+2=10$, matching. Other options give different values. Answer: (A) (4,2)	
18.	3 digits remain unknown, each ranging over 10 possibilities, giving $10\times10\times10=1000$ combinations. Probability of a correct guess = 1/1000. Answer: (A) 1/1000	1
19.	Since $2\times3=6>1$, R applies: $\tan^{-1}(2)+\tan^{-1}(3)=\pi+\tan^{-1}[(2+3)/(1-6)]=\pi+\tan^{-1}(-1)=\pi-\pi/4=3\pi/4$. Adding $\tan^{-1}(1)=\pi/4$: total $=\pi/4+3\pi/4=\pi$, confirming A. R is the exact identity used to derive this. Answer: (A) Both true, R is the correct explanation of A.	1
20.	R correctly states the centroid formula. If $\vec{a}+\vec{b}+\vec{c}=\vec{0}$, then centroid $=(\vec{a}+\vec{b}+\vec{c})/3=\vec{0}/3=\vec{0}$, i.e. the origin — confirming A, directly via R. Answer: (A) Both true, R is the correct explanation of A.	1

	SECTION B	
21.	A. $\tan^{-1}(\sqrt{3})=\pi/3$. $\tan(\pi/3-\pi/6)=\tan(\pi/6)=1/\sqrt{3}$. [2] OR B. Domain requires $\lvert 2x-3\rvert\geq1 \rightarrow 2x-3\geq1$ or $2x-3\leq-1 \rightarrow x\geq2$ or $x\leq1$. Domain $=(-\infty,1]\cup[2,\infty)$. [2]	2
22.	$dy/dx = d/dx[\log(\tan(x/2))] = [\sec^2(x/2)\cdot(1/2)]/\tan(x/2) = (1/2)\cdot[1/(\sin(x/2)\cos(x/2))] = (1/2)\cdot(2/\sin x) = 1/\sin x = \operatorname{cosec} x$ (using $\sin x=2\sin(x/2)\cos(x/2)$). Hence proved. [2]	2
23.	A. Let $f(x)=1/x$, $f'(x)=-1/x^2$. $f(x)+f'(x)=1/x-1/x^2=(x-1)/x^2$, matching the integrand. So the integral $= e^x/x+C$. [2] OR B. $x^2=4y$ meets $y=9$ at $x=\pm6$. Area $=\int_{-6}^{6}(9-x^2/4)dx=2\int_0^{6}(9-x^2/4)dx=2[9x-x^3/12]_0^{6}=2[54-18]=2(36)=72$ sq units. [2]	2
24.	$f(0)=1$ (from $x=y=0$). $f'(0)=\lim[f(h)-1]/h=4$ (given). $f'(x)=f(x)\cdot f'(0)=4f(x)$. $f'(2)=4\times f(2)=4\times5=20$. [2]	2
25.	$OA=(3,1,4)$, $OB=(5,-3,2)$. $P=$ midpoint of $AB=(4,-1,3)$. $OP=(4,-1,3)$. $OA\times OP=\lvert i\ j\ k;3\ 1\ 4;4\ -1\ 3\rvert=i(1\cdot3-4\cdot(-1))-j(3\cdot3-4\cdot4)+k(3\cdot(-1)-1\cdot4)=i(3+4)-j(9-16)+k(-3-4)=7i+7j-7k$. Magnitude $=\sqrt{49+49+49}=\sqrt{147}=7\sqrt{3}$. Area of the parallelogram $=7\sqrt{3}$ sq units. [2]	2

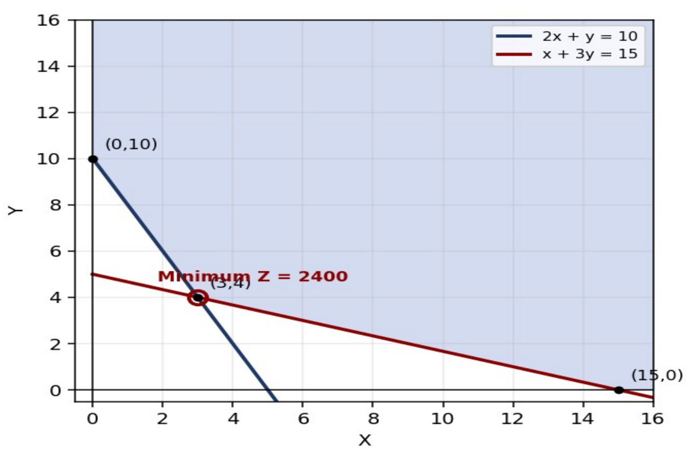
	SECTION C	
26.	A. $y(\log x+4)=4x \rightarrow y=4x/(\log x+4)$. $dy/dx=[4(\log x+4)-4x/x]/(\log x+4)^2=[4\log x+16-4]/(\log x+4)^2=4(\log x+3)/(\log x+4)^2$. At $x=e$ ($\log x=1$): $dy/dx=4(1+3)/(1+4)^2=16/25$. [3] OR B. $dx/d\theta=2a(1+\cos\theta)$, $dy/d\theta=2a\sin\theta$. $dy/dx=\sin\theta/(1+\cos\theta)=\tan(\theta/2)$. $d/d\theta[\tan(\theta/2)]=(1/2)\sec^2(\theta/2)$. Since $1+\cos\theta=2\cos^2(\theta/2)$: $d^2y/dx^2=(1/2)\sec^2(\theta/2)/[2a\cdot2\cos^2(\theta/2)]=\sec^4(\theta/2)/(8a)$. [3]	3
27.	Let $V=(4/3)\pi r^3$ be the volume and $S=4\pi r^2$ the surface area of the bubble at time t. Given $dV/dt=-kS$ for constant $k>0$ (volume decreasing). $dV/dt=4\pi r^2(dr/dt)$. So $4\pi r^2(dr/dt)=-k(4\pi r^2) \rightarrow dr/dt=-k$, a constant, independent of r. Hence the radius	3

<table>
<tr><td></td><td>decreases at a constant rate. [3]</td><td></td></tr>
<tr><td>28.</td><td>A. $\int_{-1}^{6}|x-3|dx = \int_{-1}^{3}(3-x)dx + \int_{3}^{6}(x-3)dx = [3x-x^2/2]_{-1}^{3} + [x^2/2-3x]_{3}^{6} = (9-4.5)-(-3-0.5) + (18-18)-(4.5-9) = 8 + 4.5 = 12.5$.

This represents the total area enclosed between the graph of $y=|x-3|$ and the x-axis, from $x=-1$ to $x=6$. [3]

OR B. $y^2=16x$ meets $x=4$ at $y=\pm 8$. Area$=2\int_0^4 4\sqrt{x}\,dx=8\cdot(2/3)x^{3/2}|_0^4=(16/3)\times 8=128/3$ sq units. [3]</td><td>3</td></tr>
<tr><td>29.</td><td>A. At $\mu=0$, the line passes through (6,4,-3), sharing the same x,y as the given point (6,4,11). Distance parallel to the z-axis = $|11-(-3)| = 14$ units. [3]

OR B. Line points: $(5+2\mu,\mu,3-\mu)$. Matching $x=3,y=-1$ (given through-point): $5+2\mu=3 \rightarrow \mu=-1$; $\mu=-1$ ✓ consistent. At $\mu=-1$: $z=3-(-1)=4$. Intersection point=(3,-1,4). Distance from z-axis$=\sqrt{9+1}=\sqrt{10}$ units. [3]</td><td>3</td></tr>
<tr><td>30.</td><td>Corner points: intersection of $2x+y=10$ and $x+3y=15$: from first, $y=10-2x$; sub: $x+3(10-2x)=15 \rightarrow -5x=-15 \rightarrow x=3,y=4$. So (3,4). Boundary meets axes at (15,0) [$x+3y\geq 15$ binding] and (0,10) [$2x+y\geq 10$ binding].



$Z=400x+300y$: $Z(15,0)=6000$; $Z(3,4)=1200+1200=2400$; $Z(0,10)=3000$.

Since the region is unbounded but opens away from the origin (positive coefficients), the minimum occurs at the smallest corner value.

Minimum Z = 2400, attained at (3,4). [3]</td><td>3</td></tr>
<tr><td>31.</td><td>Let $P(A)=0.7$ (Aman selected), $P(B)=p$ (Bina selected), independent.

P(exactly one) $= 0.7(1-p)+0.3p = 0.7-0.4p = 0.4 \rightarrow 0.4p=0.3 \rightarrow p=0.75$.

So P(Bina selected) = 0.75.

P(at least one) $= 1-(0.3)(0.25) = 1-0.075 = 0.925$. [3]</td><td>3</td></tr>
</table>

<table>
<tr><td colspan="3">SECTION D</td></tr>
<tr><td>32.</td><td>AB = [[1,-1,2],[3,4,-5],[2,-1,3]] × [[7,1,-3],[-19,-1,11],[-11,-1,7]]:

Row1: [1(7)+(-1)(-19)+2(-11), 1(1)+(-1)(-1)+2(-1), 1(-3)+(-1)(11)+2(7)] = [7+19-22, 1+1-2, -3-</td><td>5</td></tr>
</table>

	11+14] = [4,0,0] Row2: [3(7)+4(-19)+(-5)(-11), 3(1)+4(-1)+(-5)(-1), 3(-3)+4(11)+(-5)(7)] = [21-76+55, 3-4+5, -9+44-35] = [0,4,0] Row3: [2(7)+(-1)(-19)+3(-11), 2(1)+(-1)(-1)+3(-1), 2(-3)+(-1)(11)+3(7)] = [14+19-33, 2+1-3, -6-11+21] = [0,0,4] So AB = 4I. Hence A^{-1} = (1/4)B. The system x-y+2z=9, 3x+4y-5z=-13, 2x-y+3z=14 can be written as AX=C where $C=[9,-13,14]^T$. X = $A^{-1}C$ = (1/4)BC = (1/4)×[[7,1,-3],[-19,-1,11],[-11,-1,7]]×$[9,-13,14]^T$ Row1: 7(9)+1(-13)+(-3)(14) = 63-13-42 = 8 → x = 8/4 = 2 Row2: -19(9)+(-1)(-13)+11(14) = -171+13+154 = -4 → y = -4/4 = -1 Row3: -11(9)+(-1)(-13)+7(14) = -99+13+98 = 12 → z = 12/4 = 3 So x=2, y=-1, z=3. [5]	
33.	A. Let $I=\int_0$^(π/2) sinx/(sinx+cosx)dx. By the substitution x→π/2-x, $I=\int_0$^(π/2) cosx/(sinx+cosx)dx also. Adding both expressions for I: $2I=\int_0$^(π/2)1 dx=π/2, so I=π/4. [5] OR B. Let t=sinx, dt=cosx dx. ∫cosx/(4-$\sin^2$x)dx = ∫dt/(4-t^2) = (1/4)ln|(2+t)/(2-t)|+C = (1/4)ln|(2+sinx)/(2-sinx)|+C. [5]	5
34.	A. Integrating directly: xy = ∫x(2cosx-3x)dx = 2∫x cosx dx - 3∫x^2dx. Using integration by parts, ∫x cosx dx = x sinx+cosx. So xy = 2(xsinx+cosx)-x^3+C = 2xsinx+2cosx-x^3+C, i.e. y = 2sinx+2cosx/x-x^2+C/x. [5] OR B. dy/dx=(x^2+3y^2)/(2xy). Put y=vx: v+x(dv/dx)=(1+3v^2)/(2v) → x(dv/dx)=(1+v^2)/(2v). Separating: 2v/(1+v^2)dv=dx/x. Integrating: ln(1+v^2)=ln|x|+C → 1+v^2=Kx. Substituting v=y/x: (x^2+y^2)/x^2=Kx → $x^2+y^2=Kx^3$. Using y(1)=2: 1+4=K → K=5. Particular solution: $x^2+y^2=5x^3$. [5]	5
35.	Line 1: (2+2s,1+s,9-3s). Line 2: (3+t,4-2t,2+4t). Given the y-coordinate of intersection is 2: 1+s=2 → s=1. Point = (2+2,1+1,9-3) = (4,2,6). Check on Line 2: 3+t=4→t=1; 4-2=2 ✓; 2+4=6 ✓. Confirmed: intersection point = (4,2,6). Direction vectors: d1=(2,1,-3), d2=(1,-2,4). Perpendicular direction = d1×d2 = i(1·4-(-3)(-2)) - j(2·4-(-3)(1)) + k(2(-2)-1(1)) = i(4-6)-j(8+3)+k(-4-1) = -2i-11j-5k. Vector equation of the perpendicular line: $\vec{r} = (4\hat{i}+2\hat{j}+6\hat{k}) + \lambda(2\hat{i}+11\hat{j}+5\hat{k})$ (taking the direction with reversed sign, equivalent line). [5]	5

SECTION E		
36.	(i) Not reflexive, since no airport has a direct flight to itself (e.g. (M,M) is not in the relation). [1] (ii) Not transitive: (M,N) and (N,P) are in the relation, but (M,P) is not — sufficient to show it is not transitive. [1] (iii)(A) Ordered pairs: {(M,N),(M,O),(N,P),(O,P),(O,Q),(P,Q)}. Domain = {M,N,O,P} (airports with an outgoing flight); Range = {N,O,P,Q} (airports with an incoming flight). [2] OR (iii)(B) No, this is not a function, since M relates to both N and O (an element of the domain has more than one image), violating the definition of a function. [2]	4

37.	(i) $P(x) = R(x) - C(x) = (-0.4x^2+30x)-(0.6x^2-10x+300) = -x^2+40x-300$. [1] (ii) $P'(x) = -2x+40 = 0 \rightarrow x=20$. [1] (iii)(A) $P''(x)=-2<0$, so x=20 gives a maximum. Maximum profit P(20)=-(400)+800-300=₹100. [2] OR (iii)(B) Since the unconstrained critical point x=20 lies within the allowed range [12,28], the maximum profit within this range is still at x=20, giving ₹100. [2]	4
38.	(i) Total percentage = $0.25\times0.50 + 0.45\times0.20 + 0.30\times0.10 = 0.125+0.09+0.03 = 0.245$ = 24.5%. [2] (ii) By Bayes' theorem: P(Group1|Difficulty) = [P(Group1)×P(Difficulty|Group1)] / P(Difficulty) = $0.125/0.245 = 125/245 = 25/49 \approx 0.5102$. [2]	4