

MATHEMATICS

Code No. 041

SAMPLE QUESTION PAPER — SET 2 | CLASS XII

Time Allowed: 3 Hours

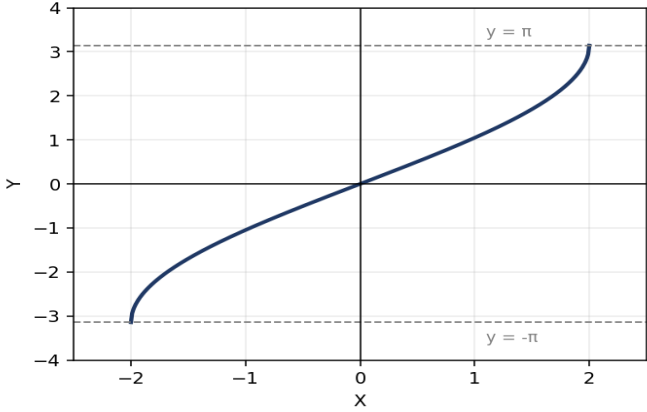
Maximum Marks: 80

General Instructions:

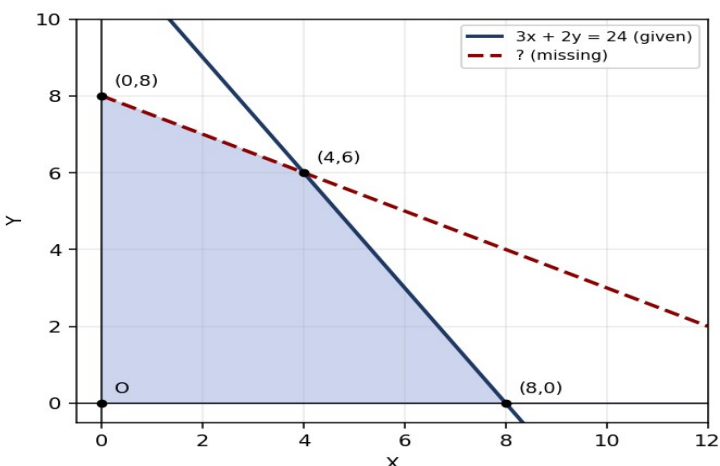
1. This question paper contains 38 questions. All questions are compulsory.
2. This question paper is divided into five Sections: A, B, C, D and E.
3. In Section A, Question numbers 1 to 18 are multiple choice questions (MCQs) with only one correct option, and Question numbers 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In Section B, Question numbers 21 to 25 are Very Short Answer (VSA) type questions, carrying 2 marks each.
5. In Section C, Question numbers 26 to 31 are Short Answer (SA) type questions, carrying 3 marks each.
6. In Section D, Question numbers 32 to 35 are Long Answer (LA) type questions, carrying 5 marks each.
7. In Section E, Question numbers 36 to 38 are Case-study based questions, carrying 4 marks each.
8. There is no overall choice. However, an internal choice has been provided in some questions in each of Sections B, C and D, and in one subpart of two questions in Section E.
9. Use of calculator is not allowed.

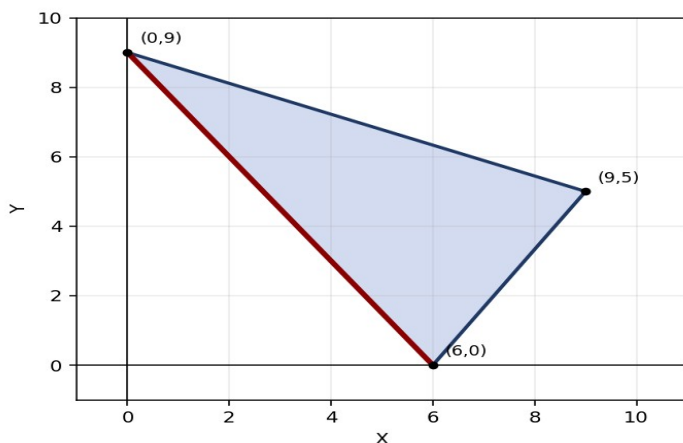
SECTION A

This section comprises multiple choice questions (MCQs) of 1 mark each (Q1-18), and Assertion-Reason questions (Q19-20).

1.	Identify the function shown in the graph:  (A) $\sin^{-1}x$ (B) $\sin^{-1}(x/2)$ (C) $2\sin^{-1}(x/2)$ (D) $\sin^{-1}(2x)$	1
2.	If for three matrices $A=[a_{ij}]_{p \times 3}$, $B=[b_{ij}]_{q \times 6}$ and $D=[d_{ij}]_{u \times v}$, the products AB and AD are both defined and are square matrices of the same order, then the values of p, q, u, v are: (A) $p=v=6$ and $q=u=3$ (B) $p=3$, $v=6$ and $q=u=6$ (C) $p=u=6$ and $q=v=3$ (D) $p=6$, $u=3$ and $q=v=6$	1
3.	If the matrix $A = \begin{bmatrix} 0 & p & -8 \\ 6 & q & r \\ s & -5 & 0 \end{bmatrix}$ is skew-symmetric, then the value of $(s+r)/(q+p)$ is: (A) $-13/6$ (B) $13/6$ (C) $-6/13$ (D) $6/13$	1

4.	<p>If A is a square matrix of order 3 and $\text{adj } A = 25$, then $A(\text{adj } A)$ is equal to:</p> <p>(A) 5 (B) 25 (C) 5I (D) 25I</p>	1
5.	<p>The inverse of the matrix $\begin{bmatrix} 7 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 9 \end{bmatrix}$ is:</p> <p>(A) $\begin{bmatrix} 1/7 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & 1/9 \end{bmatrix}$ (B) $\begin{bmatrix} 7 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 9 \end{bmatrix}$ (C) $\begin{bmatrix} -1/7 & 0 & 0 \\ 0 & -1/3 & 0 \\ 0 & 0 & -1/9 \end{bmatrix}$ (D) $\begin{bmatrix} 9 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 7 \end{bmatrix}$</p>	1
6.	<p>The value of the determinant $\begin{vmatrix} \cos 58^\circ & \sin 58^\circ \\ \sin 32^\circ & \cos 32^\circ \end{vmatrix}$ is:</p> <p>(A) 0 (B) 1/2 (C) $\sqrt{3}/2$ (D) 1</p>	1
7.	<p>If the function $f(x) = \{kx-1, x \leq 2; 3x-3, x > 2\}$ is continuous at $x=2$, the value of k is:</p> <p>(A) 1 (B) 2 (C) 3 (D) 4</p>	1
8.	<p>If $f(x) = x \tan^{-1}x$, then $f'(1)$ is equal to:</p> <p>(A) $\pi/4 + 1/2$ (B) $\pi/4 - 1/2$ (C) $-\pi/4 + 1/2$ (D) $-\pi/4 - 1/2$</p>	1
9.	<p>The function $f(x) = 15 - 3x - 2x^2$ is increasing on the interval:</p> <p>(A) $(-\infty, -3/4]$ (B) $(-\infty, 3/4)$ (C) $[-3/4, \infty)$ (D) $[-3/4, 3/4]$</p>	1
10.	<p>The differential equation $x dx + 4y dy = 0$ represents a family of:</p> <p>(A) circles (B) ellipses (C) parabolas (D) hyperbolas</p>	1
11.	<p>If $f(2a-x) = -f(x)$, then the value of $\int_0^{2a} f(x) dx$ is:</p> <p>(A) 0 (B) 2a (C) $a \int f(x) dx$ (D) $2 \int_0^a f(x) dx$</p>	1

12.	<p>If $\int x^9 \sin^2(x^{10}) \cos(x^{10}) dx = a \sin^3(x^{10}) + C$, then a is equal to:</p> <p>(A) $-1/30$ (B) $1/30$ (C) $1/10$ (D) $1/3$</p>	1
13.	<p>A car travels in a straight line given by the vector $3\hat{i}-\hat{j}+4\hat{k}$. An observer stands beside a straight track given by $\vec{r}=(1+2\lambda)\hat{i}+6\lambda\hat{j}+3\lambda\hat{k}$. The projected length of the car's path on the track is:</p> <p>(A) $12/7$ units (B) $7/12$ units (C) $12/13$ units (D) $13/12$ units</p>	1
14.	<p>The distance of the point with position vector $5\hat{i}+3\hat{j}+12\hat{k}$ from the y-axis is:</p> <p>(A) 13 units (B) $\sqrt{34}$ units (C) 5 units (D) 12 units</p>	1
15.	<p>If $\vec{a}=3\hat{i}+7\hat{j}-\hat{k}$, $\vec{b}=5\hat{i}-2\hat{j}+9\hat{k}$ and $\vec{c}=9\hat{i}+4\hat{j}-10\hat{k}$ are three given vectors, then $(3\vec{a}\cdot\hat{i})\hat{i} - (2\vec{b}\cdot\hat{j})\hat{j} + (\vec{c}\cdot\hat{k})\hat{k}$ is the same as the vector:</p> <p>(A) \vec{a} (B) \vec{b} (C) $\vec{a}-\vec{b}$ (D) \vec{c}</p>	1
16.	<p>A student comes across an incomplete question: Maximise $Z = 5x + 4y + 3$, subject to $x \geq 0, y \geq 0, 3x + 2y \leq 24, \dots$ (one constraint missing). The graph below is given for this LPP.</p>  <p>The missing constraint is:</p> <p>(A) $x + 2y \leq 16$ (B) $2x + y \geq 16$ (C) $x + 2y \geq 16$ (D) $2x + y \leq 16$</p>	1
17.	<p>The feasible region of an LPP is bounded, and the objective function $Z = 3x + 2y$ attains its minimum value at more than one point. One of these points is $(6, 0)$, as shown below.</p>	1



Then one of the other points at which Z attains its minimum value is:

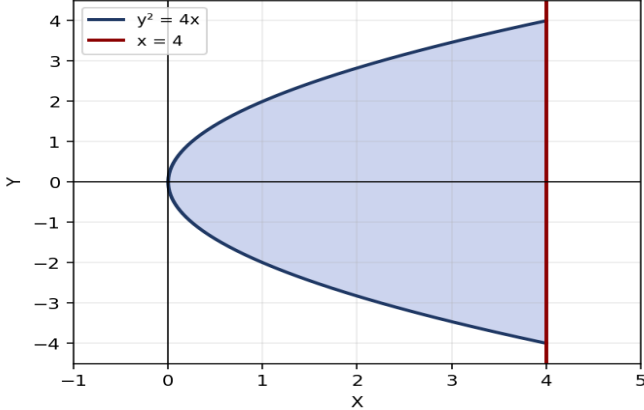
- (A) (4, 3) (B) (2, 7) (C) (5, 2) (D) (0, 0)

<p>18.</p>	<p>A person observed the first 3 digits of a friend's 4-digit locker code. What is the probability of correctly guessing the remaining digit in a single attempt?</p> <p>(A) 1/10 (B) 1/100 (C) 1/9 (D) 1</p>	<p>1</p>
<p>19.</p>	<p>Q19 and Q20 are Assertion (A) and Reason (R) based questions. Mark the correct choice as:</p> <p>(A) Both (A) and (R) are true and (R) is the correct explanation of (A). (B) Both (A) and (R) are true but (R) is not the correct explanation of (A). (C) (A) is true but (R) is false. (D) (A) is false but (R) is true.</p> <p>Assertion (A): Value of the expression $\sin^{-1}(\sqrt{3}/2) + \cos^{-1}(1) + \tan^{-1}(0)$ is $\pi/3$. Reason (R): Principal value branch of $\cos^{-1}x$ is $[0, \pi]$ and that of $\tan^{-1}x$ is $(-\pi/2, \pi/2)$.</p>	<p>1</p>
<p>20.</p>	<p>Assertion (A): If two vectors \vec{u} and \vec{v} are such that $\vec{u} + \vec{v} = \vec{u} - \vec{v}$, then \vec{u} and \vec{v} are perpendicular to each other. Reason (R): $\vec{u} + \vec{v} ^2 - \vec{u} - \vec{v} ^2 = 4(\vec{u} \cdot \vec{v})$.</p>	<p>1</p>

SECTION B

This section comprises 5 Very Short Answer (VSA) type questions of 2 marks each.

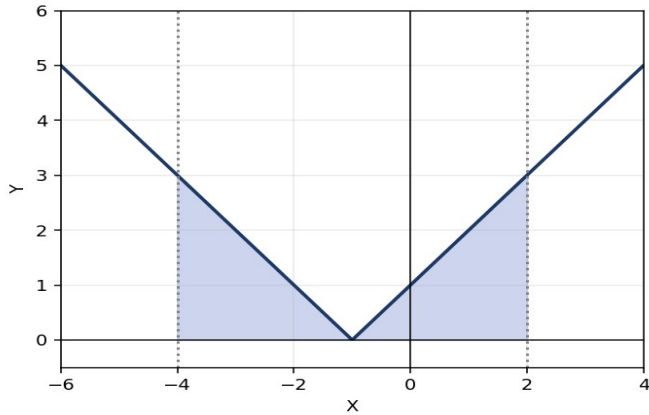
<p>21.</p>	<p>A. Evaluate: $\sin(\sin^{-1}(-1/2) + \pi/3)$</p> <p>OR</p> <p>B. Find the domain of $\cos^{-1}(3x-1)$.</p>	<p>2</p>
<p>22.</p>	<p>If $y = \log(\operatorname{cosec} x - \cot x)$, prove that $dy/dx = \operatorname{cosec} x$.</p>	<p>2</p>

23.	<p>A. Find: $\int e^x (x+1)/(x+2)^2 dx$</p> <p>OR</p> <p>B. Find the area of the shaded region enclosed by the curve $y^2 = 4x$ and the line $x = 4$.</p> 	2
24.	<p>If $f(x+y) = f(x)f(y)$ for all $x, y \in \mathbb{R}$, and $f(3) = 4$, $f'(0) = 5$, find $f'(3)$ using the definition of the derivative.</p>	2
25.	<p>The two vectors $2\hat{i}+3\hat{j}+\hat{k}$ and $6\hat{i}-\hat{j}+3\hat{k}$ represent the sides OA and OB respectively of a $\triangle OAB$, where O is the origin. P is the midpoint of AB such that OP is a median of the triangle. Find the area of the parallelogram with adjacent sides OA and OP.</p>	2

SECTION C

This section comprises 6 Short Answer (SA) type questions of 3 marks each.

26.	<p>A. If $x^y = e^{3(x-y)}$, find dy/dx in terms of x, and hence find its value at $x = e$.</p> <p>OR</p> <p>B. If $x = a(\theta - \sin\theta)$, $y = a(1 + \cos\theta)$, find d^2y/dx^2.</p>	3
27.	<p>A metal cube is being heated such that the rate of increase of its volume at any instant is directly proportional to its surface area at that instant. Show that the side of the cube increases at a constant rate.</p>	3
28.	<p>A. Sketch the graph $y = x+1$. Evaluate $\int_{-4}^2 x+1 dx$. What does this value represent on the graph?</p>	3



OR

B. Using integration, find the area of the region enclosed by the parabola $y^2 = 9x$ and the line $x = 4$.

29. A. Find the distance of the point $(5, 3, 9)$ from the line $\vec{r} = (5\hat{i} + 3\hat{j} - 2\hat{k}) + \mu(2\hat{i} + 4\hat{j} + 7\hat{k})$, measured parallel to the z-axis.

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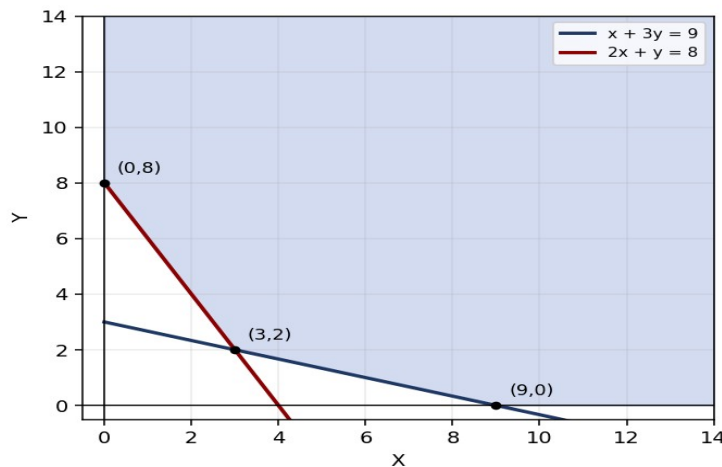
OR

B. Find the point of intersection of the line $\vec{r} = (4\hat{i} + 2\hat{k}) + \mu(\hat{i} + 2\hat{j} - \hat{k})$ and the line through $(2, -4, 7)$ parallel to the z-axis. Also find the distance of this intersection point from the z-axis.

30. Solve the following linear programming problem graphically:

3

Minimise $Z = 300x + 200y$, subject to the constraints: $x + 3y \geq 9$, $2x + y \geq 8$, $x \geq 0$, $y \geq 0$.



31. Two candidates, Neha and Priya, appear for an interview. The probability that Neha is selected is 0.6, and the probability that exactly one of them is selected is 0.5. Their selections are independent of each other. Find the probability that Priya is selected. Also find the probability that at least one of them is selected.

3

SECTION D

This section comprises 4 Long Answer (LA) type questions of 5 marks each.

32.	For two matrices $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & -1 & 1 \\ 3 & 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & -2 & -1 \\ 1 & 10 & -7 \\ 3 & 6 & -5 \end{bmatrix}$, find the product AB and hence solve the system of equations: $x + 2y - 3z = 8$; $2x - y + z = -1$; $3x + z = 2$	5
33.	A. Evaluate: $\int_0^1 x \log(1+x) dx$ OR B. Find: $\int \cos x / (1+\sin^2 x) dx$	5
34.	A. Solve the differential equation: $d/dx(xy) = x(\sin x + x^2)$ OR B. Find the particular solution of the differential equation: $(x^2-y^2)dx + 2xy dy = 0$, given that $y(1) = 1$.	5
35.	The two lines $(x-2)/1 = (y+1)/(-2) = (z-3)/2$ and $x/3 = (y+4)/1 = (z-7)/(-2)$ intersect at a point whose y-coordinate is -3. Find the coordinates of their point of intersection. Also find the vector equation of the line perpendicular to both the given lines and passing through this point of intersection.	5

SECTION E

This section comprises 3 case-study based questions of 4 marks each.

36.	<p>Case Study 1</p> <p>A sports analyst is studying match results between five cricket teams A, B, C, D and E in a tournament. The following results have been recorded:</p> <ol style="list-style-type: none"> 1. Team A defeated Team B, and Team A defeated Team C. 2. Team B defeated Team D. 3. Team C defeated Team D, and Team C defeated Team E. 4. Team D defeated Team E. <p>The analyst wants to represent and analyse this data as a relation on the set of teams. Use the given data to answer the following:</p> <p>(i) Is this relation reflexive? Justify. [1]</p> <p>(ii) Is this relation transitive? Justify. [1]</p> <p>(iii)(A) Represent the relation as a set of ordered pairs. Also state its domain and range. [2]</p> <p>OR</p> <p>(iii)(B) Does this relation represent a function from the set of teams to itself? Justify your answer. [2]</p>	4
37.	<p>Case Study 2</p> <p>A juice stall's cost of preparing x glasses of juice, and the revenue generated from selling them, are modelled as:</p> <p>$C(x) = 0.4x^2 - 9x + 130$ and $R(x) = -0.2x^2 + 15x$, where $C(x)$ and $R(x)$ are both in ₹.</p> <p>To maximise profit, the stall owner needs to analyse these functions using calculus. Use the given models to answer the following:</p> <p>(i) Derive the profit function $P(x)$. [1]</p>	4

	<p>(ii) Find the critical point of $P(x)$. [1]</p> <p>(iii)(A) Determine whether the critical point corresponds to a maximum or minimum profit, using the second derivative test. Also find this profit. [2]</p> <p>OR</p> <p>(iii)(B) If the stall's resources allow it to prepare a minimum of 12 but not more than 28 glasses per hour, identify the practical value of x that maximises profit within this range, and calculate the maximum profit. [2]</p>	
<p>38.</p>	<p>Case Study 3</p> <p>In a school, students are grouped by their average nightly sleep duration. Group 1 (less than 6 hours) makes up 20% of students, Group 2 (6 to 8 hours) makes up 60%, and Group 3 (more than 8 hours) makes up 20%. It was found that 50% of Group 1, 20% of Group 2, and 10% of Group 3 students reported difficulty concentrating in class.</p> <p>(i) What is the total percentage of students who report difficulty concentrating in class? [2]</p> <p>(ii) A student is selected at random and is found to report difficulty concentrating. What is the probability that this student belongs to Group 1 (less than 6 hours of sleep)? [2]</p>	<p>4</p>

MATHEMATICS

Code No. 041 — Marking Scheme

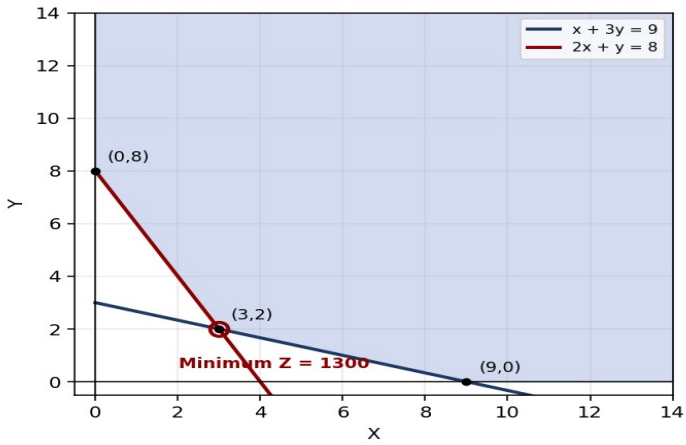
MARKING SCHEME — SET 2 | CLASS XII

SECTION A		
1.	The curve is bounded within $x \in [-2, 2]$ and has range bounded by $y = \pi$ and $y = -\pi$, matching $2\sin^{-1}(x/2)$ (since \sin^{-1} itself ranges over $[-\pi/2, \pi/2]$ with domain $[-1, 1]$, scaled by factor 2 in both). Answer: (C) $2\sin^{-1}(x/2)$	1
2.	Columns of A = rows of B: $3=q$. AB is $p \times 6$; square $\rightarrow p=6$. Columns of A = rows of D: $3=u$. AD is $p \times v = 6 \times v$; square (same order 6×6) $\rightarrow v=6$. So $p=v=6$, $q=u=3$. Answer: (A)	1
3.	Skew-symmetric: diagonal=0 $\rightarrow q=0$. $a_{12}=p, a_{21}=6 \rightarrow p=-6$. $a_{13}=-8, a_{31}=s \rightarrow s=8$. $a_{23}=r, a_{32}=-5 \rightarrow r=5$. $(s+r)/(q+p) = (8+5)/(0-6) = 13/(-6) = -13/6$. Answer: (A) $-13/6$	1
4.	$ \text{adj}A = A ^{(n-1)}$; $n=3 \rightarrow A ^2=25 \rightarrow A =5$. $A(\text{adj}A) = A I = 5I$. Answer: (C) $5I$	1
5.	Inverse of a diagonal matrix is the diagonal matrix of reciprocals of each entry. Answer: (A) $[[1/7, 0, 0], [0, 1/3, 0], [0, 0, 1/9]]$	1
6.	$\cos 58^\circ \cos 32^\circ - \sin 58^\circ \sin 32^\circ = \cos(58^\circ + 32^\circ) = \cos 90^\circ = 0$. Answer: (A) 0	1
7.	At $x=2$: $2k-1 = 6-3=3 \rightarrow 2k=4 \rightarrow k=2$. Answer: (B) 2	1
8.	$f(x) = \tan^{-1}x + x/(1+x^2)$. $f'(1) = \pi/4 + 1/2$. Answer: (A) $\pi/4 + 1/2$	1
9.	$f'(x) = -3-4x \geq 0 \rightarrow x \leq -3/4$. Answer: (A) $(-\infty, -3/4]$	1
10.	Integrating: $x^2/2 + 2y^2 = C$, a family of ellipses. Answer: (B) ellipses	1
11.	Substituting $x \rightarrow 2a-x$ in $\int_0^{2a} f(x) dx$ and using $f(2a-x) = -f(x)$ shows the integral equals its own negative, so it must be 0. Answer: (A) 0	1
12.	Let $u = x^{10}, du = 10x^9 dx$. Integral = $(1/10) \int \sin^2 u \cos u du = (1/10)(\sin^3 u/3) + C = \sin^3 u/30 + C$. $a = 1/30$. Answer: (B) $1/30$	1
13.	Direction of track = $2\hat{i} + 6\hat{j} + 3\hat{k}$. $\vec{a} \cdot \vec{b} = 3(2) + (-1)(6) + 4(3) = 6 - 6 + 12 = 12$. $ \vec{b} = \sqrt{4+36+9} = \sqrt{49} = 7$. Projection = $12/7$. Answer: (A) $12/7$ units	1
14.	Distance from y-axis = $\sqrt{x^2 + z^2} = \sqrt{25 + 144} = \sqrt{169} = 13$. Answer: (A) 13 units	1
15.	$3(\vec{a} \cdot \hat{i}) = 3(3) = 9 \rightarrow 9\hat{i}$. $-2(\vec{b} \cdot \hat{j}) = -2(-2) = 4 \rightarrow 4\hat{j}$. $(\vec{c} \cdot \hat{k}) = -10 \rightarrow -10\hat{k}$. Sum = $9\hat{i} + 4\hat{j} - 10\hat{k} = \vec{c}$. Answer: (D) \vec{c}	1
16.	From the graph, the boundary meets the x-axis at (8,0) rather than (0,0)-to-(8,0)-only from $3x+2y=24$ (which alone would cross at (0,8))... checking: the second line $x+2y=16$ crosses the axes at (16,0) and (0,8), cutting the region so the actual vertex is (8,0) with $3x+2y=24$ binding there, then transitioning to $x+2y=16$ up to (0,8) via intersection (4,6). This matches $x+2y \leq 16$. Answer: (A) $x + 2y \leq 16$	1
17.	Since $Z = 3x + 2y$ is minimised along the entire edge from (6,0) to (0,9) (parallel to lines of constant	1

	Z), any point on this edge gives the same minimum. Checking (4,3): $Z=3(4)+2(3)=12+6=18$, matching $Z(6,0)=18$. Other options give different values. Answer: (A) (4,3)	
18.	Only 1 digit (out of 10 possibilities, 0-9) is unknown. Probability of a correct single guess = 1/10. Answer: (A) 1/10	1
19.	$\sin^{-1}(\sqrt{3}/2)=\pi/3$. $\cos^{-1}(1)=0$. $\tan^{-1}(0)=0$. $\text{Sum}=\pi/3+0+0=\pi/3$, matching Assertion (A) — A is true. R correctly states the principal branches of \cos^{-1} and \tan^{-1} , but omits any mention of \sin^{-1} 's branch (which was also used in evaluating A), so R does not fully/directly explain A. Answer: (B) Both true, R is not the correct explanation of A.	1
20.	Expanding: $ \vec{u}+\vec{v} ^2= \vec{u} ^2+2\vec{u}\cdot\vec{v}+ \vec{v} ^2$ and $ \vec{u}-\vec{v} ^2= \vec{u} ^2-2\vec{u}\cdot\vec{v}+ \vec{v} ^2$. Their difference is $4(\vec{u}\cdot\vec{v})$, confirming R. If $ \vec{u}+\vec{v} = \vec{u}-\vec{v} $, their squares are equal, so $4(\vec{u}\cdot\vec{v})=0$, giving $\vec{u}\cdot\vec{v}=0$, i.e. $\vec{u}\perp\vec{v}$ — confirming A, directly explained by R. Answer: (A) Both true, R is the correct explanation of A.	1

SECTION B		
21.	A. $\sin^{-1}(-1/2)=-\pi/6$. $\sin(-\pi/6+\pi/3)=\sin(\pi/6)=1/2$. [2] OR B. Domain requires $-1\leq 3x-1\leq 1 \rightarrow 0\leq 3x\leq 2 \rightarrow 0\leq x\leq 2/3$. Domain=[0,2/3]. [2]	2
22.	$dy/dx = d/dx[\log(\operatorname{cosec}x-\cot x)] = (-\operatorname{cosec}x\cdot\cot x + \operatorname{cosec}^2x)/(\operatorname{cosec}x-\cot x) = \operatorname{cosec}x(\operatorname{cosec}x-\cot x)/(\operatorname{cosec}x-\cot x) = \operatorname{cosec}x$. Hence proved. [2]	2
23.	A. Let $f(x)=1/(x+2)$, $f'(x)=-1/(x+2)^2$. $f(x)+f'(x)=1/(x+2)-1/(x+2)^2=(x+1)/(x+2)^2$, matching the integrand. So the integral = $e^x/(x+2)+C$. [2] OR B. $y^2=4x$ meets $x=4$ at $y=\pm 4$. $\text{Area}=\int_{-4}^4(4-y^2/4)dy=2\int_0^4(4-y^2/4)dy=2[4y-y^3/12]_0^4=2[16-16/3]=2(32/3)=64/3$ sq units. [2]	2
24.	$f(0)=1$ (from $x=y=0$). $f'(0)=\lim_{h\rightarrow 0}[f(h)-1]/h=5$ (given). $f'(x)=\lim_{h\rightarrow 0}[f(x+h)-f(x)]/h=f(x)\cdot\lim_{h\rightarrow 0}[f(h)-1]/h=f(x)\cdot f'(0)=5f(x)$. $f(3)=5\times f(3)=5\times 4=20$. [2]	2
25.	$OA=(2,3,1)$, $OB=(6,-1,3)$. P=midpoint of $AB=(4,1,2)$. $OP=(4,1,2)$. $OA\times OP= i\ j\ k; 2\ 3\ 1; 4\ 1\ 2 =i(3\cdot 2-1\cdot 1)-j(2\cdot 2-1\cdot 4)+k(2\cdot 1-3\cdot 4)=i(6-1)-j(4-4)+k(2-12)=5i+0j-10k$. Magnitude= $\sqrt{(25+0+100)}=\sqrt{125}=5\sqrt{5}$. Area of the parallelogram= $5\sqrt{5}$ sq units. [2]	2

SECTION C		
26.	A. $y \log x=3(x-y) \rightarrow y(\log x+3)=3x \rightarrow y=3x/(\log x+3)$. $dy/dx=[3(\log x+3)-3x/x]/(\log x+3)^2=[3\log x+9-3]/(\log x+3)^2=3(\log x+2)/(\log x+3)^2$. At $x=e$ ($\log x=1$): $dy/dx=3(1+2)/(1+3)^2=9/16$. [3] OR B. $dx/d\theta=a(1-\cos\theta)$, $dy/d\theta=-a \sin\theta$. $dy/dx=-\sin\theta/(1-\cos\theta)=-\cot(\theta/2)$. $d/d\theta[-\cot(\theta/2)]=(1/2)\operatorname{cosec}^2(\theta/2)$. Since $1-\cos\theta=2\sin^2(\theta/2)$: $d^2y/dx^2=(1/2)\operatorname{cosec}^2(\theta/2)/[a\cdot 2\sin^2(\theta/2)]=(1/4a)\operatorname{cosec}^4(\theta/2)$. [3]	3
27.	Let $V=s^3$ be the volume and $S=6s^2$ the surface area of the cube at time t. Given $dV/dt=kS$ for constant	3

	<p>$k > 0$.</p> <p>$dV/dt = 3s^2(ds/dt)$. So $3s^2(ds/dt) = k(6s^2) \rightarrow ds/dt = 2k$, a constant, independent of s. Hence the side increases at a constant rate. [3]</p>	
28.	<p>A. $\int_{-4}^2 x+1 dx = \int_{-4}^{-1} -(x+1) dx + \int_{-1}^2 (x+1) dx = [-x^2/2 - x]_{-4}^{-1} + [x^2/2 + x]_{-1}^2 = (-0.5+1) - (-8+4) + (4+2) - (0.5-1) = 4.5+4.5 = 9$.</p> <p>This represents the total area enclosed between the graph of $y= x+1$ and the x-axis, from $x=-4$ to $x=2$. [3]</p> <p>OR B. $y^2=9x$ meets $x=4$ at $y=\pm 6$. Area $= 2 \int_0^4 3\sqrt{x} dx = 6 \cdot (2/3)x^{3/2} \Big _0^4 = 4 \times 8 = 32$ sq units. [3]</p>	3
29.	<p>A. At $\mu=0$, the line passes through $(5,3,-2)$, sharing the same x,y as the given point $(5,3,9)$. Distance parallel to the z-axis $= 9-(-2) = 11$ units. [3]</p> <p>OR B. Line points: $(4+\mu, 2\mu, 2-\mu)$. Matching $x=2, y=-4$ (of the given through-point): $4+\mu=2 \rightarrow \mu=-2$; $2\mu=-4 \rightarrow \mu=-2$ ✓ consistent. At $\mu=-2$: $z=2-(-2)=4$. Intersection point $= (2, -4, 4)$. Distance from z-axis $= \sqrt{(4+16)} = \sqrt{20} = 2\sqrt{5}$ units. [3]</p>	3
30.	<p>Corner points: intersection of $x+3y=9$ and $2x+y=8$: from first, $x=9-3y$; sub: $2(9-3y)+y=8 \rightarrow 18-5y=8 \rightarrow y=2, x=3$. So $(3,2)$. Boundary meets axes at $(9,0)$ [$x+3y \geq 9$ binding] and $(0,8)$ [$2x+y \geq 8$ binding].</p>  <p>$Z = 300x + 200y$: $Z(9,0) = 2700$; $Z(3,2) = 900 + 400 = 1300$; $Z(0,8) = 1600$.</p> <p>Since the region is unbounded but opens away from the origin (positive coefficients), the minimum occurs at the smallest corner value.</p> <p>Minimum $Z = 1300$, attained at $(3,2)$. [3]</p>	3
31.	<p>Let $P(N) = 0.6$ (Neha selected), $P(P) = p$ (Priya selected), independent.</p> <p>$P(\text{exactly one}) = 0.6(1-p) + 0.4p = 0.6 - 0.2p = 0.5 \rightarrow 0.2p = 0.1 \rightarrow p = 0.5$.</p> <p>So $P(\text{Priya selected}) = 0.5$.</p> <p>$P(\text{at least one}) = 1 - (0.4)(0.5) = 1 - 0.2 = 0.8$. [3]</p>	3

SECTION D

32.	<p>$AB = [[1,2,-3],[2,-1,1],[3,0,1]] \times [[-1,-2,-1],[1,10,-7],[3,6,-5]]$:</p>	5
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	<p>Row1: $[1(-1)+2(1)+(-3)(3), 1(-2)+2(10)+(-3)(6), 1(-1)+2(-7)+(-3)(-5)] = [-1+2-9, -2+20-18, -1-14+15] = [-8,0,0]$</p> <p>Row2: $[2(-1)+(-1)(1)+1(3), 2(-2)+(-1)(10)+1(6), 2(-1)+(-1)(-7)+1(-5)] = [-2-1+3, -4-10+6, -2+7-5] = [0,-8,0]$</p> <p>Row3: $[3(-1)+0(1)+1(3), 3(-2)+0(10)+1(6), 3(-1)+0(-7)+1(-5)] = [-3+0+3, -6+0+6, -3+0-5] = [0,0,-8]$</p> <p>So $AB = -8I$. Hence $A^{-1} = -(1/8)B$.</p> <p>The system $x+2y-3z=8, 2x-y+z=-1, 3x+z=2$ can be written as $AX=C$ where $C=[8,-1,2]^T$.</p> <p>$X = A^{-1}C = -(1/8)BC = -(1/8) \times [[-1,-2,-1],[1,10,-7],[3,6,-5]] \times [8,-1,2]^T$</p> <p>Row1: $-1(8)+(-2)(-1)+(-1)(2) = -8+2-2 = -8 \rightarrow x = -(-8)/8 = 1$</p> <p>Row2: $1(8)+10(-1)+(-7)(2) = 8-10-14 = -16 \rightarrow y = -(-16)/8 = 2$</p> <p>Row3: $3(8)+6(-1)+(-5)(2) = 24-6-10 = 8 \rightarrow z = -(8)/8 = -1$</p> <p>So $x=1, y=2, z=-1$. [5]</p>	
33.	<p>A. Using integration by parts with $u=\log(1+x), dv=x dx: v=x^2/2, du=dx/(1+x)$.</p> <p>Integral = $(x^2/2)\log(1+x) _0^1 - \int_0^1 (x^2/2)/(1+x) dx = (1/2)\log 2 - (1/2)\int_0^1 x^2/(1+x) dx$.</p> <p>Since $x^2/(1+x) = x-1+1/(1+x): \int_0^1 [x-1+1/(1+x)]dx = [x^2/2-x+\log(1+x)]_0^1 = (0.5-1+\log 2) = \log 2 - 0.5$.</p> <p>So the integral = $(1/2)\log 2 - (1/2)(\log 2 - 0.5) = 0.25 = 1/4$. [5]</p> <p>OR B. Let $t=\sin x, dt=\cos x dx. \int \cos x/(1+\sin^2 x)dx = \int dt/(1+t^2) = \tan^{-1}(t)+C = \tan^{-1}(\sin x)+C$. [5]</p>	5
34.	<p>A. Integrating directly: $xy = \int x \sin x dx + \int x^3 dx$. Using integration by parts, $\int x \sin x dx = -x \cos x + \sin x$. So $xy = -x \cos x + \sin x + x^4/4 + C$, i.e. $y = -\cos x + \sin x/x + x^3/4 + C/x$. [5]</p> <p>OR B. $dy/dx = (y^2 - x^2)/(2xy)$. Put $y=vx: v+x(dv/dx) = (v^2-1)/(2v) \rightarrow x(dv/dx) = -(1+v^2)/(2v)$. Separating: $2v/(1+v^2)dv = -dx/x$. Integrating: $\ln(1+v^2) = -\ln x + C \rightarrow x(1+v^2) = K$.</p> <p>Substituting $v=y/x: x+y^2/x = K \rightarrow x^2+y^2 = Kx$. Using $y(1)=1: 1+1=K \rightarrow K=2$. Particular solution: $x^2+y^2=2x$. [5]</p>	5
35.	<p>Line 1: $(2+s, -1-2s, 3+2s)$. Line 2: $(3t, -4+t, 7-2t)$.</p> <p>Given the y-coordinate of intersection is -3: $-1-2s=-3 \rightarrow s=1$. Point = $(2+1, -3, 3+2) = (3, -3, 5)$.</p> <p>Check on Line 2: $3t=3 \rightarrow t=1; -4+1=-3 \checkmark; 7-2=5 \checkmark$. Confirmed: intersection point = $(3, -3, 5)$.</p> <p>Direction vectors: $d_1=(1, -2, 2), d_2=(3, 1, -2)$. Perpendicular direction = $d_1 \times d_2 = i((-2)(-2)-2(1)) - j((1)(-2)-2(3)) + k((1)(1)-(-2)(3)) = i(4-2) - j(-2-6) + k(1+6) = 2i+8j+7k$.</p> <p>Vector equation of the perpendicular line: $\vec{r} = (3\hat{i} - 3\hat{j} + 5\hat{k}) + \lambda(2\hat{i} + 8\hat{j} + 7\hat{k})$. [5]</p>	5

SECTION E

36.	<p>(i) Not reflexive, since no team has defeated itself (e.g. (A,A) is not in the relation). [1]</p> <p>(ii) Not transitive: (A,B) and (B,D) are in the relation, but (A,D) is not — sufficient to show it is not transitive. [1]</p> <p>(iii)(A) Ordered pairs: $\{(A,B),(A,C),(B,D),(C,D),(C,E),(D,E)\}$. Domain = $\{A,B,C,D\}$ (teams with at least one win); Range = $\{B,C,D,E\}$ (teams with at least one loss). [2]</p> <p>OR (iii)(B) No, this is not a function, since A relates to both B and C (an element of the domain has more than one image), violating the definition of a function. [2]</p>	4
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<p>37.</p>	<p>(i) $P(x) = R(x) - C(x) = (-0.2x^2 + 15x) - (0.4x^2 - 9x + 130) = -0.6x^2 + 24x - 130$. [1]</p> <p>(ii) $P'(x) = -1.2x + 24 = 0 \rightarrow x = 20$. [1]</p> <p>(iii)(A) $P''(x) = -1.2 < 0$, so $x = 20$ gives a maximum. Maximum profit $P(20) = -0.6(400) + 24(20) - 130 = -240 + 480 - 130 = ₹110$. [2]</p> <p>OR (iii)(B) Since the unconstrained critical point $x = 20$ lies within the allowed range $[12, 28]$, the maximum profit within this range is still at $x = 20$, giving ₹110. [2]</p>	<p>4</p>
<p>38.</p>	<p>(i) Total percentage = $0.20 \times 0.50 + 0.60 \times 0.20 + 0.20 \times 0.10 = 0.10 + 0.12 + 0.02 = 0.24 = 24\%$. [2]</p> <p>(ii) By Bayes' theorem: $P(\text{Group1} \text{Difficulty}) = [P(\text{Group1}) \times P(\text{Difficulty} \text{Group1})] / P(\text{Difficulty}) = 0.10 / 0.24 = 10/24 = 5/12 \approx 0.4167$. [2]</p>	<p>4</p>