MATHEMATICS (STANDARD)

Code No. 041

SAMPLE QUESTION PAPER — SET 1 | CLASS X

Time Allowed: 3 Hours **Maximum Marks: 80**

General Instructions:

1. This question paper contains 38 questions. All questions are compulsory.
2. The paper is divided into five Sections: A, B, C, D and E.
3. In Section A, Question numbers 1 to 18 are multiple choice questions (MCQs) and question numbers 19 and 20 are Assertion-Reason based questions, of 1 mark each.
4. In Section B, Question numbers 21 to 25 are Very Short Answer (VSA) type questions, carrying 2 marks each.
5. In Section C, Question numbers 26 to 31 are Short Answer (SA) type questions, carrying 3 marks each.
6. In Section D, Question numbers 32 to 35 are Long Answer (LA) type questions, carrying 5 marks each.
7. In Section E, Question numbers 36 to 38 are case-study based questions carrying 4 marks each, with sub-parts of 1, 1 and 2 marks respectively.
8. There is no overall choice. However, an internal choice has been provided in 2 questions of Section B, 2 questions of Section C and 2 questions of Section D. An internal choice is provided in all 2-mark sub-parts of Section E.
9. Draw neat figures wherever required. Take $\pi = 22/7$ wherever required, unless stated otherwise.
10. Use of calculators is not permitted.

SECTION A *Section A consists of 20 questions of 1 mark each.*		
1.	If $a = 2^3 \times 3^2 \times 5$, $b = 2^2 \times 3^3 \times 5^2$ and $c = 2 \times 3^2 \times 5^3$, then HCF(a, b, c) equals (A) 30 (B) 60 (C) 90 (D) 180	1
2.	The largest number which divides 70 and 125, leaving remainders 5 and 8 respectively, is (A) 11 (B) 12 (C) 13 (D) 15	1
3.	If α, β are the zeroes of $x^2 - 6x + 8$, then $\alpha^2 + \beta^2$ equals (A) 16 (B) 20 (C) 28 (D) 36	1
4.	The value of k for which $kx^2 - 6x - 2 = 0$ has no real roots is (A) $k > -4.5$ (B) $k < -4.5$ (C) $k = -4.5$ (D) $k \geq -4.5$	1
5.	If $S_n = 2n^2 + 3n$ for an AP, then its first term and common difference are (A) a = 3, d = 2 (B) a = 5, d = 4 (C) a = 4, d = 5 (D) a = 2, d = 3	1
6.	The value of k for which the system $kx + 3y = k - 3$, $12x + ky = k$ has no solution is (A) 6 (B) −6 (C) 4 (D) −4	1
7.	If A(2, 3), B(4, k) and C(6, −3) are collinear, the value of k is (A) −1 (B) 0 (C) 1 (D) 2	1
8.	The point (−4, 6) divides the segment joining A(−6, 10) and B(3, −8) in the ratio	1

	(A) 2 : 3 (B) 2 : 7 (C) 3 : 7 (D) 3 : 2	
9.	D is on BC of △ABC such that ∠ADC = ∠BAC. If CA = 12 cm, CB = 15 cm, then CD equals (A) 8 cm (B) 9 cm (C) 9.6 cm (D) 10 cm	1
10.	△ABC ~ △PQR; perimeters are 32 cm and 48 cm. If PQ = 18 cm, then AB equals (A) 10 cm (B) 12 cm (C) 13.5 cm (D) 14 cm	1
11.	PA, PB are tangents from external point P; if ∠APB = 70°, then ∠AOB equals (A) 90° (B) 100° (C) 110° (D) 120°	1
12.	If tanA + cotA = 2, then $\tan^2 A + \cot^2 A$ equals (A) 0 (B) 2 (C) 4 (D) 6	1
13.	(1 + tanθ + secθ)(1 + cotθ − cosecθ) equals (A) 0 (B) 1 (C) 2 (D) secθ	1
14.	The angle of elevation of the top of a tower from a point 30√3 m away is 30°. The height of the tower is (A) 15 m (B) 20 m (C) 30 m (D) 30√3 m	1
15.	The area of the largest circle that can be drawn inside a square of side 14 cm is (A) 77 cm^2 (B) 154 cm^2 (C) 196 cm^2 (D) 308 cm^2	1
16.	Two cubes of volume 64 cm^3 each are joined end to end. The surface area of the resulting cuboid is (A) 144 cm^2 (B) 160 cm^2 (C) 176 cm^2 (D) 192 cm^2	1
17.	For a distribution, mean = 25 and mode = 22. Using the empirical relation, its median is (A) 22 (B) 23 (C) 24 (D) 25	1
18.	Two dice are thrown together. The probability of getting a sum of 9 is (A) 1/12 (B) 1/9 (C) 1/6 (D) 5/36	1
19.	Assertion (A): The HCF of two consecutive natural numbers is always 1. Reason (R): Two consecutive natural numbers are always co-prime. (A) Both A and R are true, and R is the correct explanation of A. (B) Both A and R are true, but R is not the correct explanation of A. (C) A is true but R is false. (D) A is false but R is true.	1
20.	Assertion (A): The quadratic equation $2x^2 - 4x + 3 = 0$ has no real roots. Reason (R): A quadratic equation $ax^2 + bx + c = 0$ has no real roots if $b^2 - 4ac < 0$. (A) Both A and R are true, and R is the correct explanation of A. (B) Both A and R are true, but R is not the correct explanation of A.	1

	(C) A is true but R is false. (D) A is false but R is true.	

SECTION B

Section B consists of 5 questions of 2 marks each.

21.	(A) Prove that $3 + 2\sqrt{5}$ is an irrational number, given that $\sqrt{5}$ is irrational. **OR** (B) Find the largest number that divides 245 and 1029, leaving remainder 5 in each case.	2
22.	Prove that $(\text{cosec}\theta - \cot\theta)^2 = (1 - \cos\theta) / (1 + \cos\theta)$.	2
23.	$\triangle ABC \sim \triangle DEF$; area($\triangle ABC$) = 64 cm^2, area($\triangle DEF$) = 121 cm^2, EF = 15.4 cm. Find BC.	2
24.	(A) Find the area of a sector of a circle of radius 21 cm whose arc length is 44 cm. **OR** (B) A chord of a circle of radius 15 cm subtends an angle of 60° at the centre. Find the area of the corresponding minor segment. (Use $\sqrt{3} = 1.73$, $\pi = 3.14$)	2
25.	The sum of the 4th and 8th terms of an AP is 24, and the sum of the 6th and 10th terms is 44. Find the first three terms of the AP.	2

SECTION C

Section C consists of 6 questions of 3 marks each.

26.	Find the zeroes of the polynomial $4x^2 - 4x - 3$ and verify the relationship between the zeroes and coefficients.	3
27.	(A) Find the mean of the following data using the step-deviation method: Class: 0–20, 20–40, 40–60, 60–80, 80–100 Frequency: 7, 12, 15, 10, 6 **OR** (B) Find the median of the following distribution: Class: 0–10, 10–20, 20–30, 30–40, 40–50, 50–60 Frequency: 8, 10, 12, 22, 30, 18	3
28.	Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that $\angle PTQ = 2\angle OPQ$.	3
29.	(A) Prove that $(\sin\theta + \text{cosec}\theta)^2 + (\cos\theta + \sec\theta)^2 = 7 + \tan^2\theta + \cot^2\theta$. **OR** (B) If $\tan A = n \cdot \tan B$ and $\sin A = m \cdot \sin B$, prove that $\cos^2 A = (m^2 - 1) / (n^2 - 1)$.	3
30.	Two dice are thrown simultaneously. Find the probability that (i) the sum of the numbers is a prime number, (ii) the product of the numbers is a perfect square, (iii) the sum is greater than 10.	3

31.	(A) A two-digit number is such that the product of its digits is 18. When 63 is subtracted from the number, the digits interchange their places. Find the number. **OR** (B) The sum of the numerator and denominator of a fraction is 7. If 1 is added to both, the fraction becomes 1/2. Find the fraction.	3

SECTION D *Section D consists of 4 questions of 5 marks each.*		
32.	A train covers a distance of 360 km at a uniform speed. Had the speed been 5 km/h more, it would have taken 1 hour less for the journey. Find the speed of the train.	5
33.	(A) Prove that in a right triangle, the square of the hypotenuse equals the sum of the squares of the other two sides, using the concept of similar triangles. Hence, in a right triangle right-angled at B, if AB = 9 cm and BC = 12 cm, find the length of the altitude from B to the hypotenuse AC. **OR** (B) Prove that the ratio of the areas of two similar triangles equals the square of the ratio of their corresponding sides. Hence, if the areas of two similar triangles are 100 cm^2 and 49 cm^2, and the largest side of the smaller triangle is 14 cm, find the largest side of the larger triangle.	5
34.	(A) A tent is in the shape of a cylinder surmounted by a cone. The height and diameter of the cylindrical part are 3 m and 14 m, and the slant height of the conical part is 10 m. Find the cost of canvas needed at ₹80 per m^2. ($\pi = 22/7$) **OR** (B) A solid consisting of a cone standing on a hemisphere is placed upright in a cylinder full of water, touching the bottom. The cylinder has radius 6 cm and height 15 cm; the hemisphere has radius 6 cm and the cone has height 8 cm. Find the volume of water left in the cylinder. ($\pi = 22/7$)	5
35.	The following table gives the observed lifetimes (in hours) of 100 electrical components: Lifetime (hrs): 0–50, 50–100, 100–150, 150–200, 200–250, 250–300 Frequency: 6, 14, 28, 32, 14, 6 Find the modal lifetime and the mean lifetime of the components.	5

SECTION E *Section E consists of 3 case-study based questions of 4 marks each.*		
36.	A city planner is designing a triangular park with vertices A(1, 1), B(7, 3) and C(3, 7) (units in km). A circular fountain is planned at the centroid. Based on the above, answer the following: (i) Find the coordinates of the centroid of $\triangle ABC$. [1] (ii) Find the length of side AB. [1] (iii) (A) Show that $\triangle ABC$ is isosceles, by finding the lengths of all three sides. [2] **OR** (iii) (B) Find the point which divides BC internally in the ratio 3 : 1 from B. [2]	4

37.	A construction company stacks pipes in a triangular pattern. The bottom-most layer has 24 pipes, and each layer above has 2 fewer pipes than the layer below, down to a top layer of just 2 pipes. Based on the above, answer the following: (i) How many layers are there in total? [1] (ii) Find the number of pipes in the 5th layer from the bottom. [1] (iii) (A) Find the total number of pipes used in the stack. [2] **OR** (iii) (B) If each pipe costs ₹250, find the total cost of all the pipes in the stack. [2]	4
38.	Two observers, standing on either side of a 50 m tall temple on the same horizontal line as its base, observe the angle of elevation of the top of the temple to be 60° and 30° respectively. Based on the above, answer the following: (i) Find the distance of the first observer (angle 60°) from the base of the temple. [1] (ii) Find the distance of the second observer (angle 30°) from the base of the temple. [1] (iii) (A) Find the distance between the two observers, assuming they are on opposite sides of the temple. [2] **OR** (iii) (B) If both observers are on the same side of the temple, find the distance between them. [2]	4

MATHEMATICS (STANDARD)

Code No. 041 — Marking Scheme

MARKING SCHEME — SET 1 | CLASS X

SECTION A		
1.	Min exponents: $2^1, 3^2, 5^1 \Rightarrow$ HCF = 2×9×5 = 90. Answer: (C) 90	1
2.	70−5=65, 125−8=117. HCF(65,117)=13. Answer: (C) 13	1
3.	$\alpha+\beta=6$, $\alpha\beta=8$. $\alpha^2+\beta^2 = 36-16 = 20$. Answer: (B) 20	1
4.	D = 36+8k < 0 ⇒ k < −4.5. Answer: (B) k < −4.5	1
5.	$a_1=S_1=5$. $a_2=S_2-S_1=14-5=9 \Rightarrow d=4$. Answer: (B) a=5, d=4	1
6.	$a_1/a_2=b_1/b_2 \Rightarrow k^2=36 \Rightarrow k=\pm6$; checking c-ratio shows k=−6 gives no solution. Answer: (B) −6	1
7.	Equal slopes: (k−3)/2 = (−3−k)/2 ⇒ k=0. Answer: (B) 0	1
8.	Using section formula and solving: ratio = 2 : 7. Answer: (B) 2 : 7	1
9.	△CAD ~ △CBA (AA) ⇒ CA/CB = CD/CA ⇒ CD = CA^2/CB = 144/15 = 9.6 cm. Answer: (C) 9.6 cm	1
10.	Ratio = 32/48 = 2/3. AB = 18×2/3 = 12 cm. Answer: (B) 12 cm	1
11.	OAPB angles sum to 360° with 2 right angles ⇒ ∠AOB = 180−70 = 110°. Answer: (C) 110°	1
12.	$(\tan A+\cot A)^2 = \tan^2 A+2+\cot^2 A = 4 \Rightarrow \tan^2 A+\cot^2 A = 2$. Answer: (B) 2	1
13.	Standard identity value = 2. Answer: (C) 2	1
14.	tan30° = h/(30√3) ⇒ h = 30√3×(1/√3) = 30 m. Answer: (C) 30 m	1
15.	r=7 cm, area = (22/7)(49) = 154 cm^2. Answer: (B) 154 cm^2	1
16.	Side a=4 cm (a^3=64). Cuboid 8×4×4. TSA = 2(32+16+32) = 160 cm^2. Answer: (B) 160 cm^2	1
17.	Mode = 3Median−2Mean ⇒ 22 = 3M−50 ⇒ M=24. Answer: (C) 24	1
18.	Sum=9: (3,6)(4,5)(5,4)(6,3) = 4/36 = 1/9. Answer: (B) 1/9	1
19.	Consecutive naturals always co-prime, so HCF=1; R correctly explains A. Answer: (A)	1
20.	D=16−24=−8<0, so no real roots; R states the exact valid criterion. Answer: (A)	1

SECTION B		
21.	(A) Assume $3+2\sqrt{5} = p/q$ (rational). Then $\sqrt{5} = (p/q-3)/2$, a rational number — contradicting $\sqrt{5}$ irrational. Hence $3+2\sqrt{5}$ is irrational. [2] OR (B) 245−5=240; 1029−5=1024. $240=2^4\times3\times5$, $1024=2^{10}$. HCF = 2^4 = 16. [2]	2
22.	LHS = $((1-\cos\theta)/\sin\theta)^2 = (1-\cos\theta)^2/(1-\cos^2\theta) = (1-\cos\theta)^2/[(1-\cos\theta)(1+\cos\theta)] = (1-\cos\theta)/(1+\cos\theta)$ = RHS. [2]	2
23.	$64/121 = (BC/15.4)^2 \Rightarrow BC/15.4 = 8/11 \Rightarrow BC = 11.2$ cm. [2]	2
24.	(A) Area = $\frac{1}{2}\times r\times l = \frac{1}{2}\times21\times44 = 462$ cm². [2] OR (B) Sector area = (60/360)(3.14)(225) = 117.75 cm². Triangle area = $(\sqrt{3}/4)(225) \approx 97.31$ cm². Minor segment = 117.75−97.31 ≈ 20.44 cm². [2]	2
25.	a+5d=12 and a+7d=22. Subtracting: 2d=10 ⇒ d=5, a=−13. First three terms: −13, −8, −3. [2]	2

SECTION C		
26.	$4x^2-4x-3 = (2x-3)(2x+1)$. Zeroes: 3/2, −1/2. [1.5] Sum = 1 = −(−4)/4 ✓. Product = −3/4 = −3/4 ✓. [1.5]	3
27.	(A) Midpoints 10,30,50,70,90; A=50,h=20. u: −2,−1,0,1,2. fu: −14,−12,0,10,12. Σfu=−4. Mean = 50+20(−4/50) = 48.4. [3] OR (B) CF: 8,18,30,52,82,100. n/2=50. Median class 30–40 (cf=30,f=22). Median = 30+[(50−30)/22]×10 ≈ 39.09. [3]	3
28.	TP = TQ (tangents from external point), so △TPQ is isosceles ⇒ ∠TPQ = ∠TQP. OP ⊥ TP (radius ⊥ tangent) ⇒ ∠OPT = 90°. So ∠OPQ = 90° − ∠TPQ. In △TPQ: ∠PTQ + 2∠TPQ = 180° ⇒ ∠TPQ = 90° − ∠PTQ/2. So ∠OPQ = 90° − (90°−∠PTQ/2) = ∠PTQ/2 ⇒ ∠PTQ = 2∠OPQ. [3]	3
29.	(A) LHS = $\sin^2\theta+2+\csc^2\theta+\cos^2\theta+2+\sec^2\theta = 1+4+(1+\cot^2\theta)+(1+\tan^2\theta) = 7+\tan^2\theta+\cot^2\theta$ = RHS. [3] OR (B) From sinA=m sinB: $\sin^2A=m^2\sin^2B$. From tanA=n tanB: sinA cosB = n cosA sinB ⇒ $\sin^2A\cos^2B = n^2\cos^2A\sin^2B$. Substitute $\sin^2A=m^2\sin^2B$: $m^2\cos^2B = n^2\cos^2A \Rightarrow \cos^2B = n^2\cos^2A/m^2$. Also $\cos^2B = 1-\sin^2B = 1-\sin^2A/m^2 = (m^2-1+\cos^2A)/m^2$. Equating: $n^2\cos^2A = m^2-1+\cos^2A \Rightarrow \cos^2A(n^2-1)=m^2-1 \Rightarrow \cos^2A=(m^2-1)/(n^2-1)$. [3]	3
30.	Total outcomes = 36. (i) Prime sums {2,3,5,7,11}: ways = 1+2+4+6+2 = 15. P = 15/36 = 5/12. (ii) Perfect-square products: (1,1)(2,2)(3,3)(4,4)(5,5)(6,6)(1,4)(4,1) = 8. P = 8/36 = 2/9. (iii) Sum>10: (5,6)(6,5)(6,6) = 3. P = 3/36 = 1/12. [1 each]	3

31.	(A) $xy=18$, and $10x+y-63=10y+x \Rightarrow x-y=7$. Solving $y^2+7y-18=0 \Rightarrow y=2, x=9$. Number = 92. [3] OR (B) x+2 over $(7-x)+2 = 1/2$... solved: $x=2$, denominator=5. Fraction = 2/5. [3]	3

SECTION D		
32.	Let speed = x. $360/x - 360/(x+5) = 1 \Rightarrow 360(x+5)-360x = x(x+5) \Rightarrow 1800 = x^2+5x$. $x^2+5x-1800=0 \Rightarrow x = [-5+\sqrt{(25+7200)}]/2 = [-5+85]/2 = 40$. Speed of the train = 40 km/h. [5]	5
33.	(A) [Proof: In right △ABC right-angled at B, draw BD⊥AC. △ABD~△CBD~△CAB by AA similarity. From △ABD~△CAB: $AB^2=AD \cdot AC$; from △CBD~△CAB: $CB^2=CD \cdot AC$. Adding: $AB^2+CB^2 = AC(AD+CD)=AC^2$.] Application: $AC = \sqrt{(81+144)} = 15$ cm. $BD = (AB\times BC)/AC = (9\times 12)/15 = 7.2$ cm. [5] OR (B) [Proof: similar triangles have proportional sides and equal heights ratio equal to side ratio; area ratio = $(\text{side ratio})^2$ follows from ½×base×height for each.] Application: $100/49=(\text{side ratio})^2 \Rightarrow$ ratio=10/7. Largest side of larger △ = 14×10/7 = 20 cm. [5]	5
34.	(A) r=7. CSA(cylinder)=$2\pi rh=2(22/7)(7)(3)=132$ m². CSA(cone)=$\pi rl=(22/7)(7)(10)=220$ m². Total=352 m². Cost = 352×80 = ₹28,160. [5] OR (B) Vol(cylinder)=$\pi(6)^2(15)=11880/7$ cm³. Vol(cone)=$(1/3)\pi(6)^2(8)=2112/7$ cm³. Vol(hemisphere)=$(2/3)\pi(6)^3=3168/7$ cm³. Water left = $11880/7 - (2112/7+3168/7) = 6600/7 \approx 942.86$ cm³. [5]	5
35.	Modal class 150–200 (f=32). Mode = $150+[(32-28)/(64-28-14)]\times 50 = 150+9.09 = 159.09$ hrs. Mean (step-deviation): midpoints 25,75,125,175,225,275; A=175,h=50. u:−3,−2,−1,0,1,2. fu: −18,−28,−28,0,14,12. Σfu=−48. Mean = $175+50(-48/100) = 151$ hrs. [5]	5

SECTION E		
36.	(i) Centroid = ((1+7+3)/3,(1+3+7)/3) = (11/3, 11/3). [1] (ii) $AB = \sqrt{(6^2+2^2)} = \sqrt{40} = 2\sqrt{10}$ km. [1] (iii)(A) $BC = \sqrt{((-4)^2+4^2)} = 4\sqrt{2}$ km. $CA = \sqrt{((-2)^2+(-6)^2)} = \sqrt{40} = 2\sqrt{10}$ km. Since $AB=CA=2\sqrt{10}$, △ABC is isosceles. [2] OR (iii)(B) Point dividing BC in ratio 3:1 from B = ((3×3+1×7)/4,(3×7+1×3)/4) = (4, 6). [2]	4
37.	(i) a=24,d=−2,last term=2. $2=24-2(n-1) \Rightarrow n=12$ layers. [1] (ii) $a_5 = 24+4(-2) = 16$ pipes. [1] (iii)(A) $S_{12} = (12/2)(24+2) = 156$ pipes. [2] OR (iii)(B) Total pipes = 156 (as above). Cost = 156×250 = ₹39,000. [2]	4
38.	(i) $\tan 60° = 50/d_1 \Rightarrow d_1 = 50/\sqrt{3} = 50\sqrt{3}/3$ m. [1]	4

	(ii) $\tan 30° = 50/d_2 \Rightarrow d_2 = 50\sqrt{3}$ m. [1] (iii)(A) (Opposite sides) Distance = $d_1+d_2 = 50\sqrt{3}/3+50\sqrt{3} = 200\sqrt{3}/3$ m. [2] OR (iii)(B) (Same side) Distance = $d_2-d_1 = 50\sqrt{3}-50\sqrt{3}/3 = 100\sqrt{3}/3$ m. [2]	