NCERT EXEMPLAR

CHAPTER 12

SURFACE AREAS AND VOLUMES

Class 10 | Complete Study Guide — All Exercises Solved + All Figures

Subject	Mathematics
Chapter	12 – Surface Areas and Volumes
Board	CBSE / NCERT
Level	Class 10 | Exemplar

DISCLAIMER

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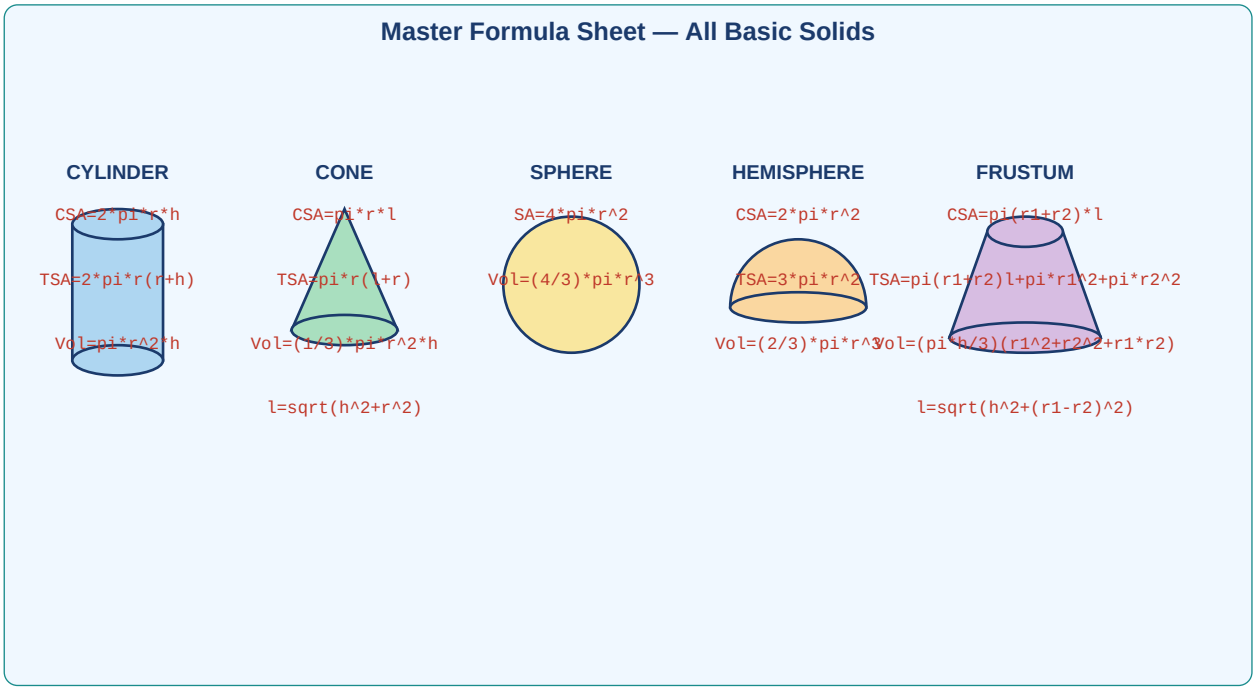
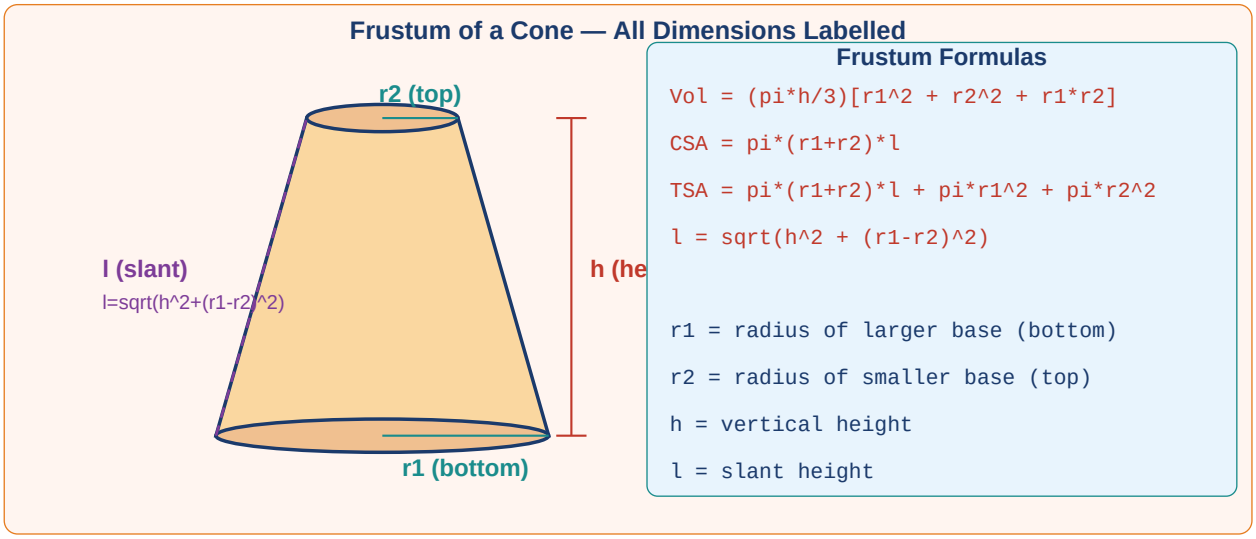
DIAGRAM 1 — Master Formula Sheet: All 5 Solids



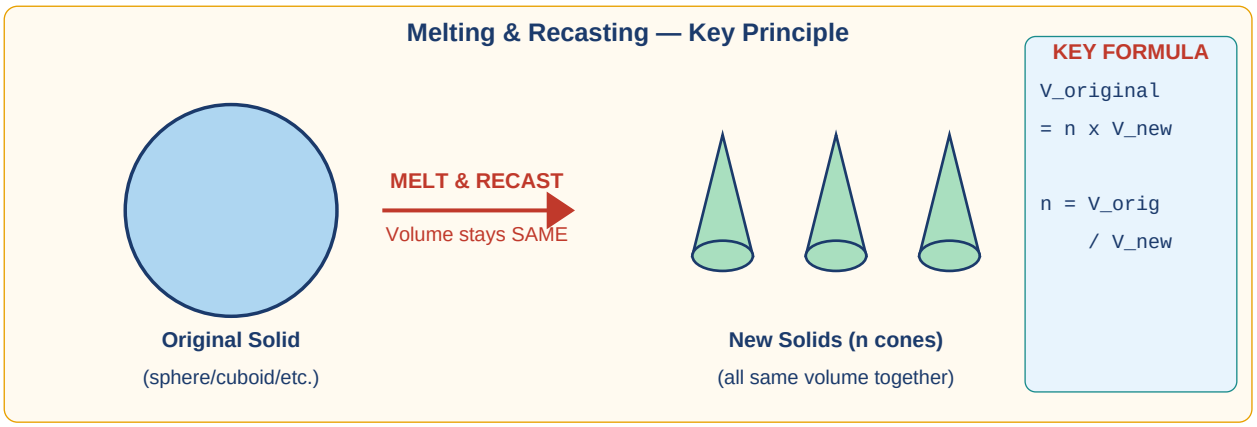
CSA=Curved, TSA=Total Surface Area, l=slant height. pi=22/7 unless stated.

DIAGRAM 2 — Frustum: All Dimensions



r1=larger base, r2=smaller base, h=height, l=slant=sqrt(h^2+(r1-r2)^2).

DIAGRAM 3 — Melting and Recasting



V_original = n x V_new. Volume is always conserved.

EXERCISE 12.1 — Multiple Choice Questions

Q1. A cylindrical pencil sharpened at one edge is the combination of:

(A) a cone and a cylinder (B) frustum of cone and cylinder (C) hemisphere and cylinder (D) two cylinders

(A) Sharpened tip = cone, body = cylinder.

Q2. A surahi is the combination of:

(A) sphere+cylinder (B) hemisphere+cylinder (C) two hemispheres (D) cylinder+cone

(A) Surahi = spherical bottom + cylindrical neck.

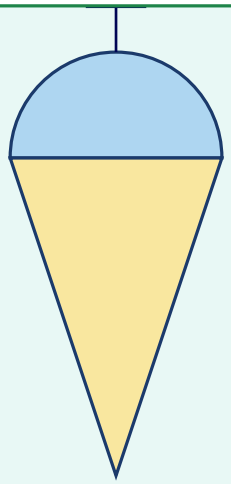
Q3. A plumbline (sahul) is the combination of:

(A) cone+cylinder (B) hemisphere+cone

(C) frustum+cylinder (D) sphere+cylinder

(B) Hemisphere + Cone

Top = hemispherical, bottom = pointed cone.



Hemisphere + Cone

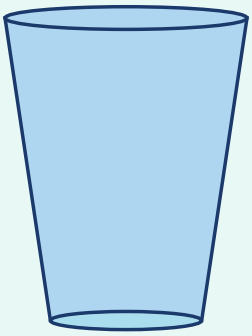
Q4. The shape of a glass (tumbler) is usually in the form of:

(A) cone (B) frustum of cone

(C) cylinder (D) sphere

(B) Frustum of a Cone

Wider at top, narrower at bottom = frustum.



Frustum of Cone

Q5. The shape of a gilli in gilli-danda is a combination of:

(A) two cylinders (B) cone+cylinder

(C) two cones+cylinder (D) two cylinders+cone

2 Cones + 1 Cylinder

(C) Two Cones + One Cylinder

Pointed at both ends (2 cones) + cylindrical middle.

Q6. A shuttlecock has the shape of:

(A) cylinder+sphere
(B) cylinder+hemisphere
(C) sphere+cone
(D) frustum+hemisphere

(D) — Hemispherical cork at base + frustum-shaped feather arrangement.

Q7. A cone cut by plane parallel to base — remaining part is called:

(A) frustum
(B) cone
(C) cylinder
(D) sphere

(A) — Cutting parallel to base and removing top gives a FRUSTUM.

Q8. Hollow cube edge 22cm, marbles dia 0.5cm. 1/8 space unfilled. Number of marbles:

(A) 142296
(B) 142396
(C) 142496
(D) 142596

(A) — Filled=(7/8)*22^3=9317cm^3. Each marble=(4/3)*(22/7)*(0.25)^3=0.06545cm^3. n=9317/0.06545=142296.

Q9. Shell (int dia 4cm, ext dia 8cm) melted to cone (base dia 8cm). Height:

(A) 12cm
(B) 14cm
(C) 15cm
(D) 18cm

(B) — Shell vol=(4/3)*pi*(4^3-2^3)=224*pi/3. Cone: (1/3)*pi*16*h=224*pi/3. h=14cm.

Q10. Cuboid 49x33x24cm moulded to sphere. Radius:

(A) 21cm
(B) 23cm
(C) 25cm
(D) 19cm

(A) — (4/3)*pi*r^3=49*33*24=38808. r^3=9261. r=21cm.

Q11. Wall 270x300x350cm, bricks 22.5x11.25x8.75cm, 1/8 mortar. Number of bricks:

(A) 11100
(B) 11200
(C) 11000
(D) 11300

(B) — Effective wall=(7/8)*28350000=24806250cm^3. Brick=2214.84cm^3. n=11200.

Q12. 12 spheres from cylinder (base dia 2cm, height 16cm). Sphere diameter:

(A) 4cm
(B) 3cm
(C) 2cm
(D) 6cm

(C) — Cyl vol=16*pi. Each sphere=(4/3)*pi*r^3=16*pi/12. r=1. Dia=2cm.

Q13. Bucket: slant h=45cm, top r=28cm, bottom r=7cm. CSA:

(A) 4950cm^2
(B) 4951
(C) 4952
(D) 4953

(A) — CSA=pi*(28+7)*45=(22/7)*35*45=4950cm^2.

Q14. Capsule: cylinder dia 0.5cm + two hemispheres, total 2cm. Capacity:

(A) 0.36
(B) 0.35
(C) 0.34
(D) 0.33 cm^3

(B) — r=0.25. Cyl length=1.5. Vol=pi*(0.25)^2*1.5+(4/3)*pi*(0.25)^3=0.35cm^3.

Q15. Two hemispheres same r joined at bases. Curved surface area:

(A) 4pi*r^2
(B) 6pi*r^2
(C) 3pi*r^2
(D) 8pi*r^2

(A) — 2 x CSA = 2 x 2*pi*r^2 = 4*pi*r^2. Flat bases hidden.

Q16. Cylinder r,h (h>2r) just encloses sphere of diameter:

(A) r
(B) 2r
(C) h
(D) 2h cm

(B) — Sphere diameter = cylinder diameter = 2r.

Q17. During conversion of shape, volume:

(A) increases
(B) decreases
(C) remains same
(D) doubles

(C) — Volume is ALWAYS conserved during melting/recasting.

Q18. Bucket diameters 44cm and 24cm, height 35cm. Capacity:

(A) 32.7 litres
(B) 33.7
(C) 34.7
(D) 31.7

(A) — r1=22,r2=12,h=35. Vol=(pi*35/3)*(484+144+264)=32706.7cm^3=32.7L.

Q19. Cross-section of cone parallel to base is a:

(A) circle
(B) frustum
(C) sphere
(D) hemisphere

(A) — Always a CIRCLE (smaller as you approach apex).

Q20. Volumes of two spheres: 64:27. Ratio of surface areas:

(A) 3:4
(B) 4:3
(C) 9:16
(D) 16:9

(D) — r1/r2=(64/27)^(1/3)=4/3. SA ratio=(4/3)^2=16:9.

EXERCISE 12.2 — True or False

Q1. Two identical solid hemispheres (r) stuck together. TSA = 6*pi*r^2.

FALSE. Only CSA visible: 2 x 2*pi*r^2 = 4*pi*r^2. Not 6*pi*r^2.

Q2. Cylinder (r,h) over same cylinder. TSA = 4*pi*r*h + 4*pi*r^2.

FALSE. Combined TSA = 4*pi*r*h + 2*pi*r^2. The two touching faces (pi*r^2 each) are hidden.

Q3. Cone (r,h) over cylinder (same r,h). TSA = pi*r[sqrt(r^2+h^2)+3r+2h].

TRUE. CSA cone + lateral cyl + bottom circle = pi*r*l+2*pi*r*h+pi*r^2 = pi*r[l+2h+r]. Correct.

Q4. Ball exactly fits in cube side a. Volume of ball = (4/3)*pi*a^3.

FALSE. r=a/2. Vol=pi*a^3/6. Not (4/3)*pi*a^3.

Q5. Vol of frustum = (1/3)*pi*h[r1^2+r2^2-r1*r2].

FALSE. Correct formula has +r1*r2 (ALL PLUS): (1/3)*pi*h[r1^2+r2^2+r1*r2].

Q6. Capacity of cylindrical vessel with hemispherical portion raised upward at bottom = (pi*r^2/3)*(3h-2r). True?

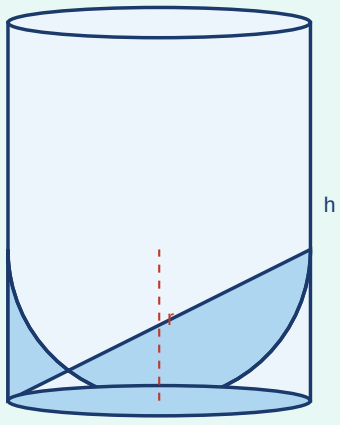
TRUE.

Vol of cylinder=pi*r^2*h.

Raised hemisphere occupies (2/3)*pi*r^3 inside.

Capacity=pi*r^2*h-(2/3)*pi*r^3

=pi*r^2*(h-2r/3)=(pi*r^2/3)*(3h-2r). Correct!



Capacity = pi*r^2*(h - 2r/3) = (pi*r^2/3)*(3h-2r)

Q7. CSA of frustum = pi*l*(r1+r2), where l=sqrt(h^2+(r1+r2)^2).

FALSE. CSA formula correct but l=sqrt(h^2+(r1-r2)^2). Difference, NOT sum.

Q8. Open metallic bucket (frustum on cylinder). Sheet area = CSA frustum + circular base + CSA cylinder.

TRUE. Covers: slant side of frustum + bottom circle of cylinder + side of cylinder. Correct!

EXERCISE 12.3 — Short Answer Questions

Q1. Three cubes edges 3,4,5cm melted into single cube. Edge?

Solution:

- 27+64+125=216cm^3. Edge=cube_root(216)=**6cm**

Q2. Shots dia 3cm from cuboid 9x11x12cm. How many?

Solution:

- Vol=1188cm^3. Each=(4/3)*(22/7)*(1.5)^3=14.14cm^3. n=1188/14.14=**84**

Q3. Bucket frustum holds 28490mL, top r=28, bottom r=21cm. Height?

Solution:

- (pi*h/3)*(784+441+588)=28490. h=28490*3*7/(22*1813)=**15cm**

Q4. Cone r=8cm, h=12cm divided at mid-point of axis parallel to base. Ratio of volumes of two parts?

Solution:

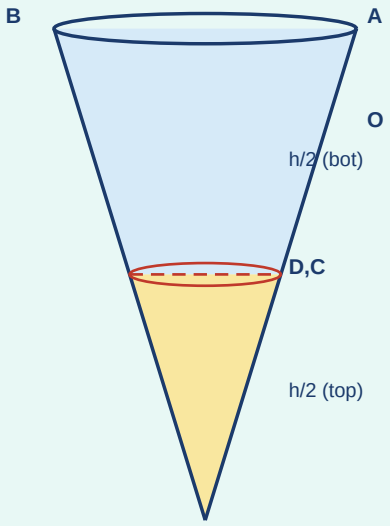
Upper cone (similar, half height): r_small=8/2=4cm, h=6cm.

Vol small cone=(1/3)*pi*16*6=32*pi

Vol full cone=(1/3)*pi*64*12=256*pi

Vol frustum=256*pi-32*pi=224*pi

Ratio = 32 : 224 = **1 : 7**



Vol ratio small cone : frustum = 1 : 7

Q5. Two identical cubes (vol 64cm^3 each) joined end to end. Surface area?

Solution:

- Edge=4cm. Cuboid=8x4x4cm. TSA=2*(32+16+32)=**160cm^2**

Q6. Cube side 7cm, conical cavity h=7cm r=3cm hollowed out. Remaining volume?

Solution:

- Cube=343. Cone=(1/3)*(22/7)*9*7=66. Remaining=**277cm^3**

Q7. Two cones r=8cm h=15cm joined at bases. Surface area?

Solution:

- l=sqrt(64+225)=17cm. TSA=2*pi*8*17=2*(22/7)*136=**855cm^2 approx**

Q8. Two cones A and B in cylindrical tube (length 21cm, r=3cm). Capacity ratio 2:1. Find heights and volumes. Also remaining cylinder volume.

Solution:

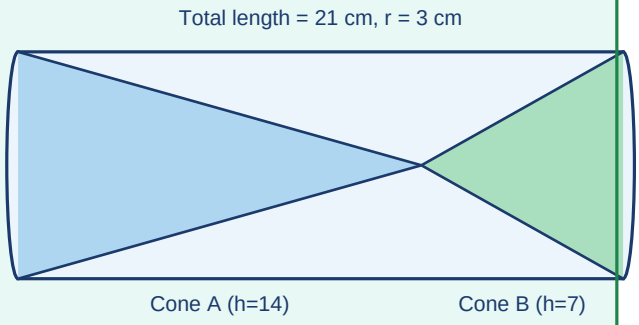
h1/h2=2, h1+h2=21 => h1=14cm, h2=7cm.

Vol A=(1/3)*pi*9*14=42*pi=132cm^3

Vol B=(1/3)*pi*9*7=21*pi=66cm^3

Cyl vol=pi*9*21=189*pi=594cm^3

Remaining=594-132-66=**396cm^3**



Q9. Ice cream cone full: r=5cm, h=10cm. 1/6 part left unfilled. Volume of ice cream?

Solution:

Shape = cone (h=10,r=5) + hemisphere on top (r=5).

Vol cone=(1/3)*pi*25*10=250*pi/3

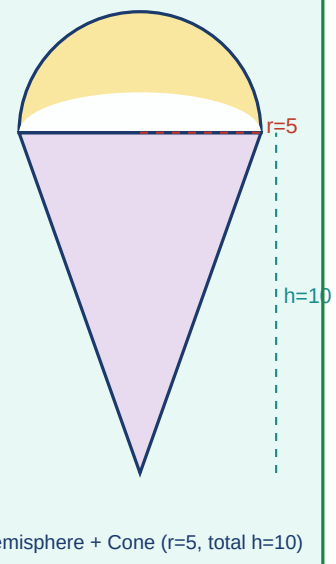
Vol hemisphere=(2/3)*pi*125=250*pi/3

Total=(500*pi/3) cm^3

Unfilled=(1/6)*cone=(1/6)*(250*pi/3)=250*pi/18

Ice cream=(500/3-250/18)*pi=2750*pi/18

=(22/7)*(2750/18) approx **479.4cm^3**



Q10. Marbles dia 1.4cm in cylinder dia 7cm. Rise 5.6cm. How many marbles?

Solution:

- Rise vol=pi*(3.5)^2*5.6=(22/7)*12.25*5.6=215.6cm^3.
- Each marble=(4/3)*(22/7)*(0.7)^3=1.437cm^3. n=215.6/1.437=**150**

Q11. Lead shots dia 4.2cm from block 66x42x21cm. How many?

Solution:

- Block=58212cm^3. Each=(4/3)*(22/7)*(2.1)^3=38.808cm^3. n=**1500**

Q12. Lead shots dia 4cm from cube edge 44cm. How many?

Solution:

- Cube=85184cm^3. Each=(4/3)*(22/7)*8=33.52cm^3. n=**2541**

Q13. Wall 24m x 0.4m x 6m, bricks 25x16x10cm, mortar=1/10. Number of bricks?

Solution:

- Wall=2400*40*600=57600000cm^3. Brick portion=(9/10)*57600000=51840000cm^3.
- One brick=4000cm^3. n=**12960**

Q14. Discs (dia 1.5cm, h 0.2cm) melted to cylinder (dia 4.5cm, h 10cm). How many?

Solution:

- Cylinder=pi*(2.25)^2*10=pi*50.625. Disc=pi*(0.75)^2*0.2=pi*0.1125. n=**450**

EXERCISE 12.4 — Long Answer Questions

Q1. Solid metallic hemisphere r=8cm melted to cone base r=6cm. Height?

Solution:

• (2/3)*pi*512=(1/3)*pi*36*h. h=1024/36=256/9 approx **28.44cm**

Q2. Rectangular tank 11x6m water 5m -> cylindrical tank r=3.5m. Height?

Solution:

• Vol=330m^3. pi*(3.5)^2*h=330. h=330/38.5=**8.57m**

Q3. Open box external 36x25x16.5cm, thickness 1.5cm. Iron needed? Weight (7.5g/cm^3)?

Solution:

• Internal: 33x22x15cm. Fe vol=14850-10890=3960cm^3.
Weight=3960*7.5=**29700g=29.7kg**

Q4. Pen barrel: 7cm long, 5mm dia. 3300 words/barrel. Words from 0.2 litre?

Solution:

• Barrel=pi*(0.25)^2*7=1.375cm^3. 200/1.375=145.45 barrels.
Words=145.45*3300=**480000**

Q5. Pipe dia 5mm, speed 10m/min. Time to fill cone dia 40cm, depth 24cm?

Solution:

• Cone vol=pi*400*8=10057cm^3. Flow=(22/7)*(0.25)^2*1000=196.4cm^3/min.
Time=51.2min=**51min 12sec**

Q6. Conical heap rice: dia 9m, h 3.5m. Volume and canvas?

Solution:

• r=4.5. l=sqrt(4.5^2+3.5^2)=5.7m. Vol=(1/3)*(22/7)*20.25*3.5=**74.25m^3**.
Canvas=pi*r*l=(22/7)*4.5*5.7=**80.61m^2**

Q7. 120000 pencils/day, length 25cm, circumference 1.5cm. Cost at Rs 0.05/dm^2?

Solution:

- CSA one=1.5*25=37.5cm^2=0.375dm^2. Total=120000*0.375=45000dm^2. Cost=45000*0.05=**Rs 2250**

Q8. Pipe dia 14cm, speed 15km/h into pond 50x44m. Time for 21cm rise?

Solution:

- Flow=pi*(0.07)^2*15000=231m^3/h. Need=50*44*0.21=462m^3. Time=**2 hours**

Q9. Iron block 4.4x2.6x1m -> hollow pipe int r=30cm, thickness 5cm. Length?

Solution:

- Vol=11.44m^3. Pipe vol=pi*(0.35^2-0.30^2)*L=pi*0.0325*L. L=**112m**

Q10. 500 persons dip, each displaces 0.04m^3, pond 80x50m. Rise?

Solution:

- Total=500*0.04=20m^3. Rise=20/4000=**0.005m=0.5cm**

Q11. 16 spheres r=2cm in box 16x8x8cm. Volume of water?

Solution:

- Box=1024. Spheres=16*(4/3)*pi*8=536.32. Water=**487.6cm^3**

Q12. Milk frustum h=16cm, r_bot=8, r_top=20cm. Cost at Rs 22/litre?

Solution:

- Vol=(pi*16/3)*(400+64+160)=10459cm^3=10.46L. Cost=10.46*22=**Rs 230.12**

Q13. Cylinder h=32cm r=18cm sand -> conical heap h=24cm. Radius and slant height?

Solution:

- pi*18^2*32=(1/3)*pi*R^2*24. R^2=1296. R=36cm. l=sqrt(1296+576)=12*sqrt(13) approx **43.27cm**

Q14. Rocket: cylinder dia 6cm h=12cm + cone slant h=5cm. TSA and volume? (pi=3.14)

Solution:

- Cone h=sqrt(25-9)=4cm. TSA=pi*r*(2h+r+l)=3.14*3*32=**301.44cm^2**.
- Vol=pi*9*12+(1/3)*pi*9*4=3.14*9*13.33=**377.1cm^3**

Q16. Hemispherical bowl r=9cm -> cylindrical bottles r=1.5cm h=4cm. How many?

Solution:

- Bowl=(2/3)*pi*729=486*pi. Bottle=pi*2.25*4=9*pi. n=486/9=**54**

Q17. Cone h=120cm r=60cm in cylinder h=180cm r=60cm (full of water). Volume water left?

Solution:

- Cyl=pi*3600*180=648000*pi. Cone=(1/3)*pi*3600*120=144000*pi.
- Water=504000*pi=(22/7)*504000=**1584000cm^3=1.584m^3**

Q18. Pipe inner r=1cm, speed 80cm/s into tank r=40cm. Rise in 30 min?

Solution:

- Vol=pi*1*80*1800=144000*pi. Rise=144000*pi/(pi*1600)=**90cm**

Q19. Roof 22x20m drains to cylinder dia 2m h=3.5m. Rainfall in cm?

Solution:

- Cyl vol=pi*1*3.5=11m^3. Depth=11/(22*20)=0.025m=**2.5cm**

Q20. Pen stand cuboid 10x5x4cm, 4 cones (r=0.5,d=2.1cm), 1 cube (edge 3cm). Wood volume?

Solution:

- Cuboid=200. 4 cones=4*(1/3)*(22/7)*0.25*2.1=2.2. Cube=27.
- Wood=200-2.2-27=**170.8cm^3**

Q15. Building = cylinder + hemispherical dome. Contains 41*19/21 m^3. Internal diameter of dome = total height. Find height.

Solution:

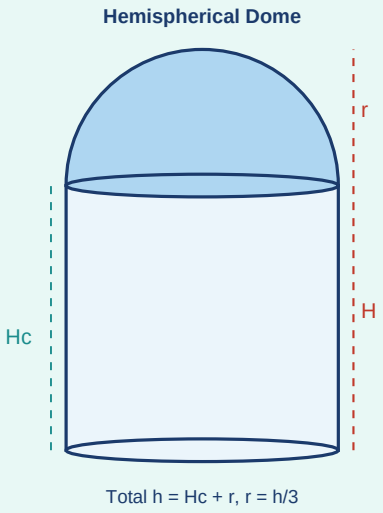
Let total height=h. Diameter=h => r=h/2.

Cyl height=h-r=h-h/2=h/2.

Vol=(2/3)*pi*(h/2)^3+pi*(h/2)^2*(h/2)

=(8/81)*pi*h^3

(8/81)*pi*h^3=880/21. h^3=64. h=**4m**



QUICK REVISION — All Key Formulas

Solid	Formulas
Cylinder (r,h)	CSA=2*pi*r*h | TSA=2*pi*r(r+h) | Vol=pi*r^2*h
Cone (r,h,l)	CSA=pi*r*l | TSA=pi*r(l+r) | Vol=(1/3)*pi*r^2*h | l=sqrt(h^2+r^2)
Sphere (r)	SA=4*pi*r^2 | Vol=(4/3)*pi*r^3
Hemisphere (r)	CSA=2*pi*r^2 | TSA=3*pi*r^2 | Vol=(2/3)*pi*r^3
Spherical shell	Vol=(4/3)*pi*(r1^3-r2^3)
Frustum (r1,r2,h,l)	CSA=pi*(r1+r2)*l | Vol=(pi*h/3)*(r1^2+r2^2+r1*r2) | l=sqrt(h^2+(r1-r2)^2)
Cuboid (l,b,h)	TSA=2(lb+bh+hl) | Vol=l*b*h | Diagonal=sqrt(l^2+b^2+h^2)
Cube (a)	TSA=6a^2 | Vol=a^3 | Diagonal=a*sqrt(3)
Melting/Recasting	V_original = n x V_new
Water flow	Vol=pi*r^2 x speed x time
Sphere in cube (side a)	r=a/2, Vol=pi*a^3/6

EXAM TIPS & COMMON MISTAKES

1. Volume is ALWAYS conserved when melting/recasting.

2. For combined solids: count only EXPOSED (visible) surfaces. Joined faces are NEVER counted.

3. Frustum slant height: l=sqrt(h^2+(r1-r2)^2) — use DIFFERENCE not sum.

4. Frustum volume: all PLUS signs — (r1^2+r2^2+r1*r2).

5. Sphere in cube side a: r=a/2, Volume=pi*a^3/6 (NOT 4*pi*a^3/3).

6. Two cubes joined: TSA=10a^2 (two touching faces disappear).