MATHEMATICS

Code No. 041

SAMPLE QUESTION PAPER — SET 1 | CLASS XII

Time Allowed: 3 Hours **Maximum Marks: 80**

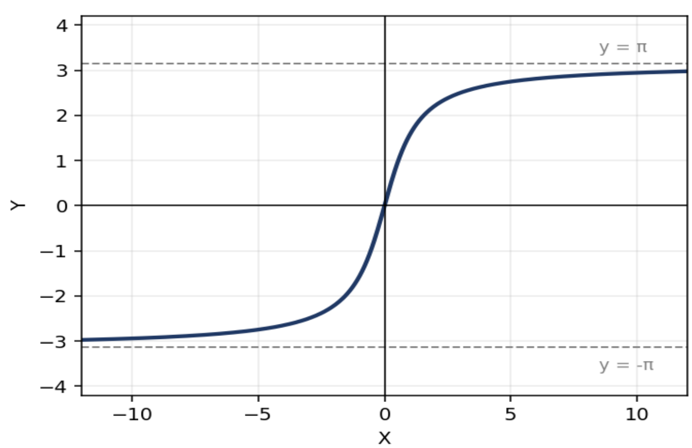
General Instructions:

1. This question paper contains 38 questions. All questions are compulsory.
2. This question paper is divided into five Sections: A, B, C, D and E.
3. In Section A, Question numbers 1 to 18 are multiple choice questions (MCQs) with only one correct option, and Question numbers 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In Section B, Question numbers 21 to 25 are Very Short Answer (VSA) type questions, carrying 2 marks each.
5. In Section C, Question numbers 26 to 31 are Short Answer (SA) type questions, carrying 3 marks each.
6. In Section D, Question numbers 32 to 35 are Long Answer (LA) type questions, carrying 5 marks each.
7. In Section E, Question numbers 36 to 38 are Case-study based questions, carrying 4 marks each.
8. There is no overall choice. However, an internal choice has been provided in some questions in each of Sections B, C and D, and in one subpart of two questions in Section E.
9. Use of calculator is not allowed.

SECTION A

This section comprises multiple choice questions (MCQs) of 1 mark each (Q1-18), and Assertion-Reason questions (Q19-20).

1. Identify the function shown in the graph: 1



(A) $\tan^{-1}x$ (B) $\tan^{-1}(2x)$ (C) $\tan^{-1}(x/2)$ (D) $2\tan^{-1}x$

2. If for three matrices $A=[a_{ij}]_{r\times 5}$, $B=[b_{ij}]_{s\times 2}$ and $D=[d_{ij}]_{t\times u}$, the products AB and AD are both defined and are square matrices of the same order, then the values of r, s, t, u are: 1

(A) $r=u=2$ and $s=t=5$

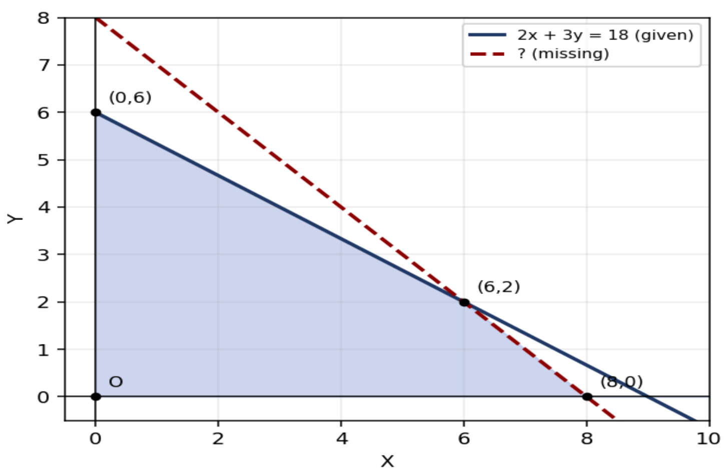
(B) $r=5$, $u=2$ and $s=t=2$

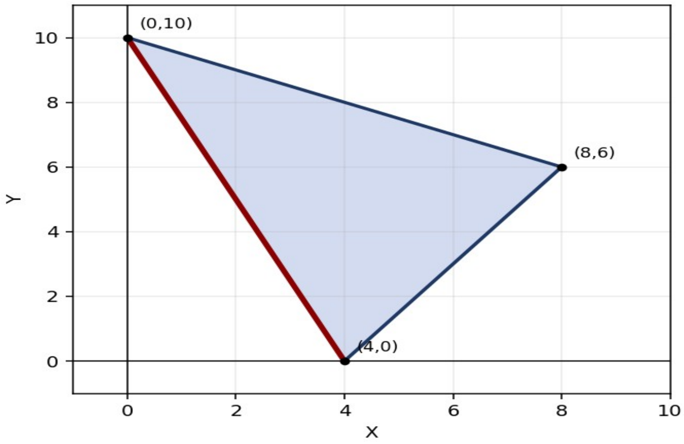
(C) $r=u=5$ and $s=t=2$

(D) $r=2$, $t=5$ and $s=u=2$

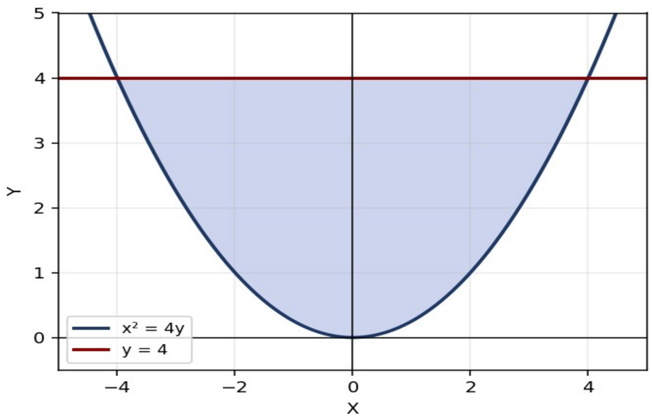
3. If the matrix A = [[0, a, -6], [4, b, c], [d, -3, 0]] is skew-symmetric, then the value of $(d+c)/(b+a)$ is: 1

	(A) -9/4 (B) 9/4 (C) -4/9 (D) 4/9	
4.	If A is a square matrix of order 4 and |adj A| = 64, then A(adj A) is equal to: (A) 4 (B) 64 (C) 4I (D) 64I	1
5.	The inverse of the matrix [[4,0,0],[0,5,0],[0,0,2]] is: (A) [[1/4,0,0],[0,1/5,0],[0,0,1/2]] (B) [[4,0,0],[0,5,0],[0,0,2]] (C) [[-1/4,0,0],[0,-1/5,0],[0,0,-1/2]] (D) [[0,0,4],[0,5,0],[2,0,0]]	1
6.	The value of the determinant | cos73° sin73° ; sin17° cos17° | is: (A) 0 (B) 1/2 (C) √3/2 (D) 1	1
7.	If the function f(x) = {kx - 2, x ≤ 3; 2x + 1, x > 3} is continuous at x = 3, the value of k is: (A) 1 (B) 2 (C) 3 (D) 4	1
8.	If f(x) = x $\cot^{-1}$x, then f'(1) is equal to: (A) π/4 - 1/2 (B) π/4 + 1/2 (C) -π/4 - 1/2 (D) -π/4 + 1/2	1
9.	The function f(x) = 12 - 2x - $3x^2$ is increasing on the interval: (A) (-∞, -1/3] (B) (-∞, 1/3) (C) [-1/3, ∞) (D) [-1/3, 1/3]	1
10.	The differential equation x dx - y dy = 0 represents a family of: (A) straight lines (B) parabolas (C) circles (D) hyperbolas	1
11.	If f(a+b-x) = f(x), then the value of $\int_a$^b (2x-a-b) f(x) dx is: (A) 0 (B) (a+b) ∫ f(x) dx	1

	(C) $(b-a)\int f(x)\,dx$ (D) $2\int x\,f(x)\,dx$	
12.	If $\int x^7 \sin^2(x^8)\cos(x^8)\,dx = a\sin^3(x^8) + C$, then a is equal to: (A) -1/24 (B) 1/24 (C) 1/8 (D) 1/3	1
13.	A bird flies through a distance in a straight line given by the vector $\hat{i} - 2\hat{j} + 2\hat{k}$. A man stands beside a straight metro track given by $\vec{r} = (2+2\lambda)\hat{i} + 3\lambda\hat{j} + 6\lambda\hat{k}$. The projected length of the flight on the metro track is: (A) 8/7 units (B) 7/8 units (C) 8/13 units (D) 13/8 units	1
14.	The distance of the point with position vector $6\hat{i} + 2\hat{j} + 8\hat{k}$ from the y-axis is: (A) 10 units (B) $\sqrt{68}$ units (C) 2 units (D) $8\sqrt{2}$ units	1
15.	If $\vec{a}=4\hat{i}+3\hat{j}-2\hat{k}$, $\vec{b}=\hat{i}-2\hat{j}+5\hat{k}$ and $\vec{c}=8\hat{i}+2\hat{j}-6\hat{k}$ are three given vectors, then $(2\vec{a}\cdot\hat{i})\hat{i} - (\vec{b}\cdot\hat{j})\hat{j} + (\vec{c}\cdot\hat{k})\hat{k}$ is the same as the vector: (A) $\vec{a}$ (B) $\vec{b}$ (C) $\vec{a}-\vec{b}$ (D) $\vec{c}$	1
16.	A student comes across an incomplete question in a book: Maximise Z = 4x + 3y + 2, subject to the constraints $x \geq 0$, $y \geq 0$, $2x + 3y \leq 18$, ... (one constraint is missing). The student finds the graph below for this LPP, and notices a constraint is missing from it.  Help the student choose the missing constraint from the graph. The missing constraint is: (A) $x + y \leq 8$ (B) $x + 2y \geq 8$ (C) $x + y \geq 8$ (D) $2x + y \leq 8$	1
17.	The feasible region of a linear programming problem is bounded, and the objective function Z = 5x + 2y attains its minimum value at more than one point. One of these points is (4, 0), as shown below.	1

	 Then one of the other points at which Z attains its minimum value is: (A) (2, 5) (B) (1, 8) (C) (3, 3) (D) (0, 0)	
18.	A thief has stolen the first 3 digits of a person's 5-digit ATM PIN (each digit from 0-9). What is the probability that the thief correctly guesses the remaining 2 digits in a single attempt? (A) 1/100 (B) 1/10 (C) 1/90 (D) 1	1
19.	Q19 and Q20 are Assertion (A) and Reason (R) based questions. Mark the correct choice as: (A) Both (A) and (R) are true and (R) is the correct explanation of (A). (B) Both (A) and (R) are true but (R) is not the correct explanation of (A). (C) (A) is true but (R) is false. (D) (A) is false but (R) is true. Assertion (A): Value of the expression $\cos^{-1}(1/2) + \sin^{-1}(1) - \text{cosec}^{-1}(1)$ is $\pi/2$. Reason (R): Principal value branch of $\sin^{-1}x$ is $[-\pi/2, \pi/2]$ and that of $\text{cosec}^{-1}x$ is $[-\pi/2, \pi/2] - \{0\}$.	1
20.	Assertion (A): Given two non-zero, non-parallel vectors $\vec{p}$ and $\vec{q}$. If $\vec{s}$ is a non-zero vector such that $\vec{s} \times (\vec{p}+\vec{q}) = \vec{0}$, then $\vec{s}$ is perpendicular to $\vec{p}\times\vec{q}$. Reason (R): The vector $(\vec{p}+\vec{q})$ is perpendicular to the plane containing $\vec{p}$ and $\vec{q}$.	1

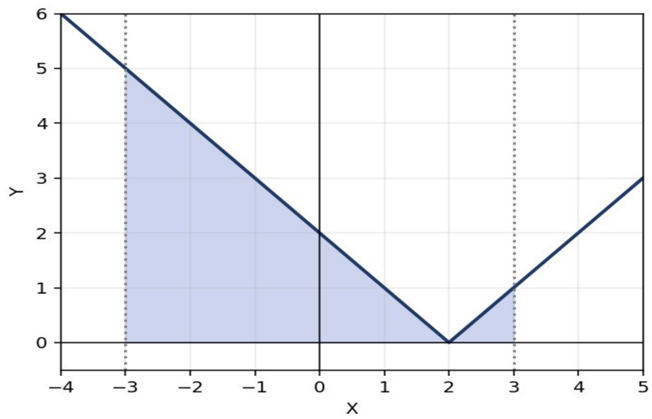
	SECTION B *This section comprises 5 Very Short Answer (VSA) type questions of 2 marks each.*	
21.	A. Evaluate: $\cos(\cos^{-1}(-1/2) + \pi/6)$ **OR** B. Find the domain of $\sin^{-1}(2x-1)$.	2
22.	If $y = \log(\sec x + \tan x)$, prove that $dy/dx = \sec x$.	2

23.	A. Find: $\int e^x \frac{(x-2)}{(x-1)^2}\,dx$ **OR** B. Find the area of the shaded region enclosed by the curve $x^2 = 4y$ and the line $y = 4$. 	2
24.	If $f(x+y) = f(x)f(y)$ for all $x, y \in R$, and $f(4) = 3$, $f'(0) = 2$, find $f'(4)$ using the definition of the derivative.	2
25.	The two vectors $2\hat{i}+\hat{j}+2\hat{k}$ and $4\hat{i}-3\hat{j}+2\hat{k}$ represent the sides OA and OB respectively of a $\triangle OAB$, where O is the origin. P is the midpoint of AB such that OP is a median of the triangle. Find the area of the parallelogram with adjacent sides OA and OP.	2

SECTION C

This section comprises 6 Short Answer (SA) type questions of 3 marks each.

26.	A. If $x^y = e^{(2(x-y))}$, find dy/dx in terms of x, and hence find its value at $x = e$. **OR** B. If $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$, find d^2y/dx^2.	3
27.	A spherical balloon is being deflated such that the rate of decrease of its volume at any instant is directly proportional to its surface area at that instant. Show that the radius of the balloon decreases at a constant rate.	3
28.	A. Sketch the graph $y = \|x - 2\|$. Evaluate $\int_{-3}^{3} \|x-2\|\,dx$. What does this value represent on the graph?	3



OR

B. Using integration, find the area of the region enclosed by the parabola $y^2 = 4x$ and the line $x = 3$.

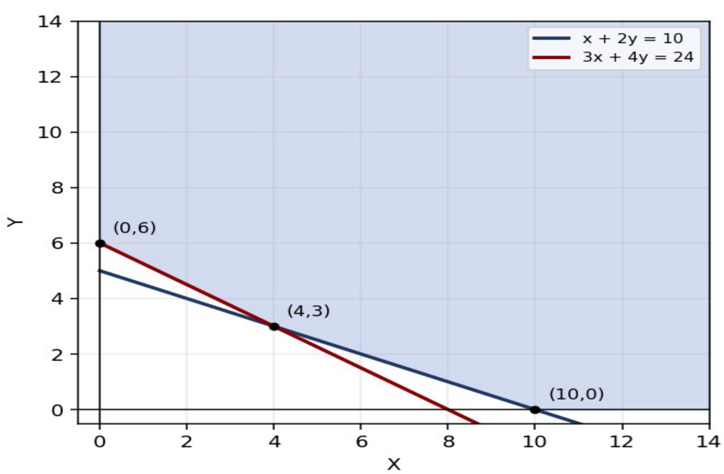
29. A. Find the distance of the point (4, 2, 7) from the line $\vec{r} = (4\hat{i}+2\hat{j}-\hat{k}) + \mu(5\hat{i}+\hat{j}+3\hat{k})$, measured parallel to the z-axis. — 3

OR

B. Find the point of intersection of the line $\vec{r} = (2\hat{i}+3\hat{k}) + \mu(\hat{i}+\hat{j}+\hat{k})$ and the line through (1,-1,4) parallel to the z-axis. Also find the distance of this intersection point from the z-axis.

30. Solve the following linear programming problem graphically: — 3

Minimise $Z = 200x + 500y$, subject to the constraints: $x + 2y \geq 10$, $3x + 4y \geq 24$, $x \geq 0$, $y \geq 0$.



31. Two candidates, Aditi and Karan, appear for an interview. The probability that Aditi is selected is 0.7, and the probability that exactly one of them is selected is 0.6. Their selections are independent of each other. Find the probability that Karan is selected. Also find the probability that at least one of them is selected. — 3

SECTION D

This section comprises 4 Long Answer (LA) type questions of 5 marks each.

32.	For two matrices A = [[1,-1,1],[2,1,-3],[1,1,1]] and B = [[4,2,2],[-5,0,5],[1,-2,3]], find the product AB and hence solve the system of equations: $x - y + z = 4$; $2x + y - 3z = 0$; $x + y + z = 2$	5
33.	A. Evaluate: $\int_0$^1 $\tan^{-1}x / (1+x^2)\,dx$ **OR** B. Find: $\int \sin x / (1+\cos^2 x)\,dx$	5
34.	A. Solve the differential equation: $d/dx(xy) = x(\cos x - x)$ **OR** B. Find the particular solution of the differential equation: $(x^2+y^2)dx - 2xy\,dy = 0$, given that $y(1) = 0$.	5
35.	The two lines $(x-1)/1 = (y+1)/2 = (z-4)/(-1)$ and $x/2 = (y-2)/(-1) = z/3$ intersect at a point whose y-coordinate is 1. Find the coordinates of their point of intersection. Also find the vector equation of the line perpendicular to both the given lines and passing through this point of intersection.	5

	SECTION E *This section comprises 3 case-study based questions of 4 marks each.*	
36.	**Case Study 1** A transport company is analysing one-way bus routes connecting five towns P, Q, R, S and T. The following direct routes have been recorded: 1. Routes from P to Q, and P to R. 2. A route from Q to S. 3. Routes from R to S, and R to T. 4. A route from S to T. The company wants to represent and analyse this data as a relation on the set of towns. Use the given data to answer the following: (i) Is this relation reflexive? Justify. [1] (ii) Is this relation transitive? Justify. [1] (iii)(A) Represent the relation as a set of ordered pairs. Also state its domain and range. [2] **OR** (iii)(B) Does this relation represent a function from the set of towns to itself? Justify your answer. [2]	4
37.	**Case Study 2** A bakery's cost of producing x cupcakes in a batch, and the revenue generated from selling them, are modelled as: $C(x) = 0.5x^2 - 10x + 200$ and $R(x) = -0.3x^2 + 22x$, where C(x) and R(x) are both in ₹. To maximise profit, the bakery needs to analyse these functions using calculus. Use the given models to answer the following: (i) Derive the profit function P(x). [1]	4

	(ii) Find the critical point of P(x). [1] (iii)(A) Determine whether the critical point corresponds to a maximum or minimum profit, using the second derivative test. Also find this profit. [2] **OR** (iii)(B) If the bakery's resources allow it to produce a minimum of 15 but not more than 25 cupcakes per hour, identify the practical value of x that maximises profit within this range, and calculate the maximum profit. [2]	
38.	**Case Study 3** In a school, students are grouped by their daily commute time to school. Group 1 (commute more than 45 minutes) makes up 25% of students, Group 2 (20 to 45 minutes) makes up 45%, and Group 3 (less than 20 minutes) makes up 30%. It was found that 40% of Group 1, 20% of Group 2, and 10% of Group 3 students reported feeling tired during morning classes. (i) What is the total percentage of students who report feeling tired during morning classes? [2] (ii) A student is selected at random and is found to report feeling tired. What is the probability that this student belongs to Group 1 (commute more than 45 minutes)? [2]	4

MATHEMATICS

Code No. 041 — Marking Scheme

MARKING SCHEME — SET 1 | CLASS XII

SECTION A		
1.	The graph shows an S-shaped curve bounded between horizontal asymptotes $y=\pi$ and $y=-\pi$, matching the range of $2\tan^{-1}x$ (since $\tan^{-1}x$ itself is bounded by $\pm\pi/2$). Answer: (D) $2\tan^{-1}x$	1
2.	For AB to be defined, columns of A = rows of B: 5=s. AB is then r×2; for square, r=2. For AD defined, columns of A=rows of D: 5=t. AD is r×u=2×u; for square (same order as AB, i.e. 2×2), u=2. So r=u=2, s=t=5. Answer: (A)	1
3.	For a skew-symmetric matrix, diagonal entries are 0 and $a_{ij}=-a_{ji}$. Diagonal: b=0. a12=a,a21=4 → a=-4. a13=-6,a31=d → d=6. a23=c,a32=-3 → c=3. $(d+c)/(b+a) = (6+3)/(0-4) = 9/(-4) = -9/4$. Answer: (A) -9/4	1
4.	$\lvert adj\ A\rvert=\lvert A\rvert^{(n-1)}$ for an n×n matrix; here n=4, so $\lvert A\rvert^3=64$ → $\lvert A\rvert=4$. $A(adj\ A)=\lvert A\rvert\cdot I=4I$. Answer: (C) 4I	1
5.	The inverse of a diagonal matrix is the diagonal matrix of reciprocals. Answer: (A) [[1/4,0,0],[0,1/5,0],[0,0,1/2]]	1
6.	Determinant = cos73°cos17° - sin73°sin17° = cos(73°+17°) = cos90° = 0. Answer: (A) 0	1
7.	Continuity at x=3 requires left value = right value: 3k-2 = 2(3)+1=7 → 3k=9 → k=3. Answer: (C) 3	1
8.	$f'(x) = \cot^{-1}x - x/(1+x^2)$. $f'(1) = \cot^{-1}(1) - 1/2 = \pi/4 - 1/2$. Answer: (A) $\pi/4 - 1/2$	1
9.	f'(x) = -2-6x. Increasing where f'(x)≥0: -2-6x≥0 → x≤-1/3. Answer: (A) (-∞,-1/3]	1
10.	Integrating: $x^2/2 - y^2/2 = C$ → $x^2-y^2=2C$, a family of hyperbolas. Answer: (D) hyperbolas	1
11.	Let I=∫(2x-a-b)f(x)dx over [a,b]. Substituting x → a+b-x: (2x-a-b) → -(2x-a-b), and f(a+b-x)=f(x) (given), so I=-I → 2I=0 → I=0. Answer: (A) 0	1
12.	Let $u=x^8$, $du=8x^7dx$. Integral = $\int(1/8)\sin^2u \cos u\, du = (1/8)(\sin^3u/3)+C = \sin^3u/24+C$. So a=1/24. Answer: (B) 1/24	1
13.	Direction vector of track = $2\hat{i}+3\hat{j}+6\hat{k}$. $\vec{a}\cdot\vec{b}$ = 1(2)+(-2)(3)+2(6) = 2-6+12=8. $\lvert\vec{b}\rvert=\sqrt{4+9+36}=\sqrt{49}=7$. Projection = 8/7. Answer: (A) 8/7 units	1
14.	Distance from the y-axis = $\sqrt{x^2+z^2} = \sqrt{36+64} = \sqrt{100} = 10$. Answer: (A) 10 units	1
15.	$\vec{a}\cdot\hat{i}=4$ (x-component of $\vec{a}$) → 2(4)=8 → $8\hat{i}$. $\vec{b}\cdot\hat{j}=-2$ (y-component of $\vec{b}$) → -(-2)=2 → $2\hat{j}$. $\vec{c}\cdot\hat{k}=-6$ (z-component of $\vec{c}$) → $-6\hat{k}$. Sum = $8\hat{i}+2\hat{j}-6\hat{k} = \vec{c}$. Answer: (D) $\vec{c}$	1
16.	From the graph, the feasible region's boundary on the x-axis ends at (8,0), not (9,0) (which is where 2x+3y=18 alone would cross). This means an additional line x+y=8 (through (8,0) and (0,8)) cuts into the region, intersecting 2x+3y=18 at (6,2). This matches x+y≤8. Answer: (A) x + y ≤ 8	1

17.	Since $Z=5x+2y$ is minimised along the entire edge from (4,0) to (0,10) (as this edge is parallel to the lines of constant Z), any point on this edge also gives the minimum. Checking (2,5): $Z=5(2)+2(5)=10+10=20$, same as $Z(4,0)=20$. The other options give different Z values. Answer: (A) (2,5)	1
18.	Only the last 2 digits are unknown, each ranging 0-9, giving $10\times10=100$ possible combinations. Probability of guessing correctly = 1/100. Answer: (A) 1/100	1
19.	$\cos^{-1}(1/2)=\pi/3$. $\sin^{-1}(1)=\pi/2$. $\operatorname{cosec}^{-1}(1)=\pi/2$ (since $\operatorname{cosec}(\pi/2)=1$, within its principal branch). Sum $= \pi/3+\pi/2-\pi/2 = \pi/3$, NOT $\pi/2$ as claimed — so Assertion (A) is false. Reason (R) correctly states the principal value branches. Answer: (D) A is false but R is true.	1
20.	A is true: $\vec{s}\times(\vec{p}+\vec{q})=\vec{0}$ means $\vec{s}$ is parallel to $(\vec{p}+\vec{q})$; since $(\vec{p}+\vec{q})$ lies in the plane of $\vec{p}$ and $\vec{q}$, and $\vec{p}\times\vec{q}$ is normal to that plane, $\vec{s}$ (parallel to a vector in the plane) is indeed perpendicular to $\vec{p}\times\vec{q}$. R is false: $(\vec{}+\vec{})$ lies IN the plane of $\vec{}\,\vec{}$, (being their sum), not perpendicular to it. Answer: (C) A is true but R is false.	1

SECTION B

21.	A. $\cos^{-1}(-1/2)=2\pi/3$. $\cos(2\pi/3+\pi/6)=\cos(5\pi/6)=-\sqrt{3}/2$. [2] OR B. Domain requires $-1\le 2x-1\le 1 \rightarrow 0\le 2x\le 2 \rightarrow 0\le x\le 1$. Domain = [0,1]. [2]	2
22.	$dy/dx = d/dx[\log(\sec x+\tan x)] = (\sec x \tan x + \sec^2 x)/(\sec x+\tan x) = \sec x(\tan x+\sec x)/(\sec x+\tan x) = \sec x$. Hence proved. [2]	2
23.	A. Let $f(x)=1/(x-1)$, $f'(x)=-1/(x-1)^2$. $f(x)+f'(x) = 1/(x-1)-1/(x-1)^2 = (x-2)/(x-1)^2$, matching the integrand. So $\int e^x[f(x)+f'(x)]dx = e^x f(x)+C = e^x/(x-1)+C$. [2] OR B. Intersection of $x^2=4y$ and $y=4$: $x^2=16\rightarrow x=\pm4$. Area $= \int_{-4}$^4 $(4-x^2/4)dx = 2\int_0^4(4-x^2/4)dx = 2[4x-x^3/12]_0$^4 $= 2[16-16/3] = 2(32/3) = 64/3$ sq units. [2]	2
24.	Setting $x=y=0$: $f(0)=f(0)^2 \rightarrow f(0)=1$ (nonzero case). $f'(0)=$lim_{h→0}$[f(h)-1]/h=2$ (given). $f'(x)=$lim_{h→0}$[f(x+h)-f(x)]/h = \lim[f(x)f(h)-f(x)]/h = f(x)\cdot\lim[f(h)-1]/h = f(x)\cdot f'(0) = 2f(x)$. $f'(4) = 2\cdot f(4) = 2\times3 = 6$. [2]	2
25.	OA=(2,1,2), OB=(4,-3,2). P is the midpoint of AB: P = ((2+4)/2, (1-3)/2, (2+2)/2) = (3,-1,2). So OP=(3,-1,2). OA × OP = |i j k; 2 1 2; 3 -1 2| = $i(1\cdot2-2\cdot(-1)) - j(2\cdot2-2\cdot3) + k(2\cdot(-1)-1\cdot3) = i(2+2) - j(4-6) + k(-2-3) = 4i+2j-5k$. $\|OA\times OP\| = \sqrt{16+4+25} = \sqrt{45} = 3\sqrt{5}$. Area of the parallelogram = $3\sqrt{5}$ sq units. [2]	2

SECTION C

26.	A. Taking log: $y \log x = 2(x-y) \rightarrow y \log x+2y=2x \rightarrow y(\log x+2)=2x \rightarrow y=2x/(\log x+2)$. Differentiating: $dy/dx = [2(\log x+2)-2x(1/x)]/(\log x+2)^2 = [2\log x+4-2]/(\log x+2)^2 = 2(\log x+1)/(\log x+2)^2$.	3

	At x=e (natural log, logx=1): $dy/dx = 2(1+1)/(1+2)^2 = 4/9$. [3] OR B. $dx/d\theta=a(1+\cos\theta)$, $dy/d\theta=a \sin\theta$. $dy/dx=\sin\theta/(1+\cos\theta)=\tan(\theta/2)$ (half-angle identity). $d/d\theta[\tan(\theta/2)]=(1/2)\sec^2(\theta/2)$. Since $1+\cos\theta=2\cos^2(\theta/2)$, $d^2y/dx^2 = [(1/2)\sec^2(\theta/2)]/[a\cdot 2\cos^2(\theta/2)] = (1/4a)\sec^4(\theta/2)$. [3]	
27.	Let $V=(4/3)\pi r^3$ be the volume and $S=4\pi r^2$ the surface area at time t. Given $dV/dt = -kS$ for some constant $k>0$ (volume decreasing). $dV/dt = 4\pi r^2(dr/dt)$. So $4\pi r^2(dr/dt) = -k(4\pi r^2) \rightarrow dr/dt = -k$, a constant, independent of r. Hence the radius decreases at a constant rate. [3]	3
28.	A. $\int_{-3}^{3}\lvert x-2\rvert dx = \int_{-3}^{2}(2-x)dx + \int_{2}^{3}(x-2)dx = [2x-x^2/2]_{-3}^{2} + [x^2/2-2x]_{2}^{3} = (4-2)-(-6-4.5) + (4.5-6)-(2-4) = 2+10.5 + (-1.5+2) = 12.5+0.5 = 13$. This value represents the total area enclosed between the graph of $y=\lvert x-2\rvert$ and the x-axis, from x=-3 to x=3. [3] OR B. $y^2=4x$ meets x=3 at $y=\pm\sqrt{12}=\pm 2\sqrt{3}$. Area $= 2\int_0^3 2\sqrt{x}\, dx = 4\int_0^3\sqrt{x}\, dx = 4\cdot(2/3)x^{3/2}\rvert_0^3 = (8/3)\cdot 3\sqrt{3} = 8\sqrt{3}$ sq units. [3]	3
29.	A. At $\mu=0$, the line passes through (4,2,-1), which shares the same x,y coordinates (4,2) as the given point (4,2,7), differing only in z. Distance parallel to the z-axis = $\lvert 7-(-1)\rvert = 8$ units. [3] OR B. The line $\vec{r} =(2\hat{i}+3\hat{k})+\mu(\hat{i}+\hat{j}+\hat{k})$ has points $(2+\mu,\mu,3+\mu)$. For x=1,y=-1 (matching the given point's x,y): $2+\mu=1 \rightarrow \mu=-1$, and $\mu=-1$ matches y=-1 too ✓. At $\mu=-1$, z=3-1=2. So intersection point = (1,-1,2). Distance from the z-axis = $\sqrt{1^2+(-1)^2}=\sqrt{2}$ units. [3]	3
30.	Corner points of the feasible region: intersection of x+2y=10 and 3x+4y=24 gives (4,3) [solving: x=10-2y, sub: 3(10-2y)+4y=24 → 30-2y=24 → y=3,x=4]. Boundary meets x-axis at (10,0) [since $x+2y\geq 10$ is more restrictive there] and y-axis at (0,6) [since $3x+4y\geq 24$ is more restrictive there].  Evaluating Z=200x+500y: Z(10,0)=2000; Z(4,3)=800+1500=2300; Z(0,6)=3000. Since the feasible region is unbounded but opens away from the origin (both coefficients of Z are positive), the minimum occurs at the corner point giving the smallest value. Minimum Z = 2000, attained at (10,0). [3]	3
31.	Let P(A)=0.7 (Aditi selected), P(K)=p (Karan selected), independent events. P(exactly one selected) = P(A)P(K')+P(A')P(K) = 0.7(1-p)+0.3p = 0.7-0.4p = 0.6 → 0.4p=0.1 → p=0.25.	3

	So P(Karan selected) = 0.25. P(at least one selected) = 1 - P(none) = 1 - (0.3)(0.75) = 1-0.225 = 0.775. [3]	

SECTION D		
32.	AB = [[1,-1,1],[2,1,-3],[1,1,1]] × [[4,2,2],[-5,0,5],[1,-2,3]]: Row1: [1(4)+(-1)(-5)+1(1), 1(2)+(-1)(0)+1(-2), 1(2)+(-1)(5)+1(3)] = [4+5+1, 2+0-2, 2-5+3] = [10, 0, 0] Row2: [2(4)+1(-5)+(-3)(1), 2(2)+1(0)+(-3)(-2), 2(2)+1(5)+(-3)(3)] = [8-5-3, 4+0+6, 4+5-9] = [0, 10, 0] Row3: [1(4)+1(-5)+1(1), 1(2)+1(0)+1(-2), 1(2)+1(5)+1(3)] = [4-5+1, 2+0-2, 2+5+3] = [0, 0, 10] So AB = 10I. Hence A^{-1} = (1/10)B. The system x-y+z=4, 2x+y-3z=0, x+y+z=2 can be written as AX=C where $C=[4,0,2]^T$. X = $A^{-1}C$ = (1/10)BC = (1/10)×[[4,2,2],[-5,0,5],[1,-2,3]]×$[4,0,2]^T$ Row1: 4(4)+2(0)+2(2)=16+0+4=20 → x=20/10=2 Row2: -5(4)+0(0)+5(2)=-20+0+10=-10 → y=-10/10=-1 Row3: 1(4)+(-2)(0)+3(2)=4+0+6=10 → z=10/10=1 So x=2, y=-1, z=1. [5]	5
33.	A. Let $u=\tan^{-1}x$, $du=dx/(1+x^2)$. When x=0,u=0; when x=1,u=π/4. Integral = $\int_0$^(π/4) u du = $[u^2/2]_0$^(π/4) = $(\pi/4)^2/2 = \pi^2/32$. [5] OR B. Let t=cosx, dt=-sinx dx. $\int \sin x/(1+\cos^2 x)dx = -\int dt/(1+t^2) = -\tan^{-1}(t)+C = -\tan^{-1}(\cos x)+C$. [5]	5
34.	A. Integrating directly: xy = ∫x(cosx-x)dx = ∫x cosx dx - $\int x^2dx$. Using integration by parts, ∫x cosx dx = x sinx+cosx. So xy = xsinx+cosx-$x^3/3$+C, i.e., y = sinx + cosx/x - $x^2/3$ + C/x. [5] OR B. This is a homogeneous equation: $dy/dx=(x^2+y^2)/(2xy)$. Put y=vx, dy/dx=v+x(dv/dx): $v+x(dv/dx)=(1+v^2)/(2v)$ → $x(dv/dx)=(1-v^2)/(2v)$. Separating: $2v/(1-v^2)dv=dx/x$. Integrating: $-\ln\|1-v^2\| =\ln\|x\|+C$ → $x(1-v^2)=K$. Substituting v=y/x: $x-y^2/x=K$ → $x^2-y^2=Kx$. Using y(1)=0: 1-0=K → K=1. Particular solution: $x^2-y^2=x$, i.e., $y^2=x^2-x$. [5]	5
35.	Line 1: (1+s, -1+2s, 4-s). Line 2: (2t, 2-t, 3t). For intersection: 1+s=2t, -1+2s=2-t, 4-s=3t. Given the y-coordinate of the intersection point is 1: -1+2s=1 → s=1. Then point = (1+1, 1, 4-1) = (2,1,3). Check on Line 2: 2t=2→t=1; 2-1=1 ✓ (matches y=1); 3(1)=3 ✓ (matches z=3). Confirmed: intersection point = (2,1,3). Direction vectors: d1=(1,2,-1), d2=(2,-1,3). Perpendicular direction = d1×d2 = i(2·3-(-1)(-1)) - j(1·3-(-1)(2)) + k(1(-1)-2(2)) = i(6-1)-j(3+2)+k(-1-4) = 5i-5j-5k, i.e., direction (1,-1,-1). Vector equation of the perpendicular line: $\vec{r} = (2\hat{i}+\hat{j}+3\hat{k}) + \lambda(\hat{i}-\hat{j}-\hat{k})$. [5]	5

SECTION E

36.	(i) Not reflexive, since no town has a direct route to itself (e.g. (P,P) is not in the relation). [1] (ii) Not transitive: (P,Q) and (Q,S) are both in the relation, but (P,S) is not — a single counterexample is enough to show the relation is not transitive. [1] (iii)(A) Ordered pairs: {(P,Q),(P,R),(Q,S),(R,S),(R,T),(S,T)}. Domain = {P,Q,R,S} (towns with an outgoing route); Range = {Q,R,S,T} (towns with an incoming route). [2] OR (iii)(B) No, this is not a function, since P is related to both Q and R (an element of the domain has more than one image), which violates the definition of a function. [2]	4
37.	(i) $P(x) = R(x)-C(x) = (-0.3x^2+22x) - (0.5x^2-10x+200) = -0.8x^2+32x-200$. [1] (ii) $P'(x) = -1.6x+32 = 0 \rightarrow x = 20$. [1] (iii)(A) $P''(x) = -1.6 < 0$, so $x=20$ gives a maximum. Maximum profit $P(20) = -0.8(400)+32(20)-200 = -320+640-200 =$ ₹120. [2] OR (iii)(B) Since the unconstrained critical point $x=20$ already lies within the allowed range [15,25], the maximum profit within this range is still at $x=20$, giving a maximum profit of ₹120. [2]	4
38.	(i) Total percentage tired = P(G1)×P(Tired|G1) + P(G2)×P(Tired|G2) + P(G3)×P(Tired|G3) = 0.25×0.40 + 0.45×0.20 + 0.30×0.10 = 0.10+0.09+0.03 = 0.22 = 22%. [2] (ii) By Bayes' theorem: P(G1|Tired) = [P(G1)×P(Tired|G1)] / P(Tired) = 0.10 / 0.22 = 10/22 = 5/11 ≈ 0.4545. [2]	4