CLASS 10 MATHEMATICS

CHAPTER 1 — REAL NUMBERS

Complete Solved Study Guide

Concepts • MCQs • Reasoning • Short & Long Answers • Exam Tips

How to Use This Guide

This guide rebuilds the entire chapter into a clean, revision-friendly format. Every question from each exercise is listed with a fully worked, step-by-step solution written in plain language. Concept boxes refresh the theory, tip boxes share shortcuts examiners love, and note boxes flag the mistakes students make most often. Work through it section by section, then revise only the highlighted boxes the night before your exam.

Part A — Key Concepts You Must Know

1. Euclid's Division Lemma

For any two positive integers a and b, you can always find exactly one pair of integers q (quotient) and r (remainder) so that:

$$a = b\,q + r, \quad \text{where } 0 \leq r < b$$

In simple words: when you divide one number by another, the remainder is always smaller than the divisor and can never be negative. This single idea powers the whole HCF algorithm below.

2. Euclid's Division Algorithm (to find HCF)

To find the HCF of two numbers c and d (with c greater than d):

- Step 1: Divide c by d and write it as $c = d \cdot q + r$.
- Step 2: If the remainder r is 0, then d is your HCF. If not, repeat the process using d and r.
- Step 3: Keep going until the remainder becomes 0. The last divisor you used is the HCF.

3. Fundamental Theorem of Arithmetic

Every composite number can be broken down into a product of prime numbers, and this prime factorisation is unique — the only thing that can change is the order in which you write the primes. For example, 60 is always $2 \times 2 \times 3 \times 5$, no matter how you reach it.

4. Primes and Squares

If a prime number p divides the square of a positive integer (p divides a^2), then that same prime p must also divide a itself. This is the key tool used in almost every 'prove it is irrational' proof.

5. Irrational Numbers — Quick Facts

- $\sqrt{2}$, $\sqrt{3}$ and $\sqrt{5}$ are irrational (their decimals never end and never repeat).
- Rational + Irrational = Irrational, and Rational − Irrational = Irrational.
- A non-zero rational multiplied or divided by an irrational stays irrational.

6. The HCF–LCM Relationship

$$\text{HCF }(a,\, b) \times \text{LCM }(a,\, b) = a \times b$$

This handy formula lets you find the LCM the moment you know the HCF (and vice versa) for any two positive integers.

7. When Does a Decimal Terminate?

Concept Recall

Write the fraction p/q in lowest terms (p and q sharing no common factor).

It TERMINATES (ends) only when q can be written using just the primes 2 and 5 — that is, $q = 2^m \times 5^n$.

If q contains any other prime (like 3 or 7), the decimal is non-terminating and repeating.

Exam Tip

Number of decimal places of a terminating fraction = the larger of the two powers m and n in $q = 2^m \times 5^n$. So $33/(2^2 \cdot 5)$ ends after 2 places because the highest power is 2.

Part B — Multiple Choice Questions (Solved)

Worked Samples

QS1. The decimal expansion of the rational number $33 / (2^2 \times 5)$ will terminate after — (A) one decimal place (B) two decimal places (C) three decimal places (D) more than 3 decimal places.

Solution

The denominator is $2^2 \times 5^1$. Since it uses only the primes 2 and 5, the decimal terminates. The number of decimal places equals the higher power, which is 2 (from 2^2). Quick check: multiply top and bottom by 5 to make the denominator 100, giving $165/100 = 1.65$ — exactly two places.

Correct Answer: (B) two decimal places

QS2. Euclid's division lemma states that for two positive integers a and b there exist unique integers q and r with $a = bq + r$, where r must satisfy — (A) $1 < r < b$ (B) $0 < r \leq b$ (C) $0 \leq r < b$ (D) $0 < r < b$.

Solution

The remainder can be as small as 0 (when b divides a perfectly) but must always stay below the divisor b. So the correct range is $0 \leq r < b$.

Correct Answer: (C) $0 \leq r < b$

Exercise 1.1

Q1. For some integer m, every even integer is of the form — (A) m (B) m + 1 (C) 2m (D) 2m + 1.

Solution

Even numbers are exactly the multiples of 2: …, −4, −2, 0, 2, 4, … Every one of them can be written as 2 times some integer m. The form 2m + 1 always gives odd numbers, so it is wrong.

Correct Answer: (C) 2m

Q2. For some integer q, every odd integer is of the form — (A) q (B) q + 1 (C) 2q (D) 2q + 1.

Solution

An odd number is always one more than an even number. Since even numbers are 2q, odd numbers are 2q + 1 (for example $5 = 2\times2 + 1$).

Correct Answer: (D) 2q + 1

Q3. $n^2 - 1$ is divisible by 8, if n is — (A) an integer (B) a natural number (C) an odd integer (D) an even integer.

Solution

Write $n^2 - 1 = (n - 1)(n + 1)$. If n is odd, then $n - 1$ and $n + 1$ are two consecutive even numbers. Among any two consecutive even numbers, one is a multiple of 4, so their product is a multiple of $4 \times 2 = 8$. Quick test: $n = 3$ gives 8 (divisible), $n = 5$ gives 24 (divisible). For an even n such as $n = 2$, we get 3, which is not divisible by 8.

Correct Answer: (C) an odd integer

> **Q4.** If the HCF of 65 and 117 is expressible in the form 65m − 117, then the value of m is — (A) 4 (B) 2 (C) 1 (D) 3.

Solution

First find the HCF. 117 = 65×1 + 52, then 65 = 52×1 + 13, then 52 = 13×4 + 0, so the HCF is 13. Now set 65m − 117 = 13, which gives 65m = 130, so m = 2.

Correct Answer: (B) 2

> **Q5.** The largest number which divides 70 and 125, leaving remainders 5 and 8 respectively, is — (A) 13 (B) 65 (C) 875 (D) 1750.

Solution

If the remainder is 5 when dividing 70, then the number divides 70 − 5 = 65 exactly. Similarly it divides 125 − 8 = 117 exactly. So the answer is the HCF of 65 and 117, which we found above to be 13.

Correct Answer: (A) 13

> **Q6.** If $a = x^3y^2$ and $b = xy^3$ where x, y are primes, then HCF (a, b) is — (A) xy (B) xy^2 (C) x^3y^3 (D) x^2y^2.

Solution

For the HCF, take each common prime to its SMALLEST power. For x: powers are 3 and 1 → take 1. For y: powers are 2 and 3 → take 2. So HCF = $x^1y^2 = xy^2$.

> **Exam Tip**
> HCF → lowest powers. LCM → highest powers. Remember it as 'HCF is the Humble one (smaller), LCM is the Large one'.

Correct Answer: (B) xy^2

> **Q7.** If $p = ab^2$ and $q = a^3b$ where a, b are primes, then LCM (p, q) is — (A) ab (B) a^2b^2 (C) a^3b^2 (D) a^3b^3.

Solution

For the LCM, take each prime to its HIGHEST power. For a: powers are 1 and 3 → take 3. For b: powers are 2 and 1 → take 2. So LCM = a^3b^2.

Correct Answer: (C) a^3b^2

> **Q8.** The product of a non-zero rational and an irrational number is — (A) always irrational (B) always rational (C) rational or irrational (D) one.

Solution

Multiplying any irrational number by a non-zero rational can never 'clean up' the irrational part. For example, $2 \times \sqrt{3} = 2\sqrt{3}$ is still irrational. The result is always irrational.

Correct Answer: (A) always irrational

> **Q9.** The least number divisible by all the numbers from 1 to 10 (both inclusive) is — (A) 10 (B) 100 (C) 504 (D) 2520.

Solution

This is just the LCM of 1 to 10. Take the highest power of each prime that appears up to 10: 2^3 (from 8), 3^2 (from 9), 5 (from 5) and 7 (from 7). Multiply: $8 \times 9 \times 5 \times 7 = 2520$.

Correct Answer: (D) 2520

> **Q10.** The decimal expansion of 14587 / 1250 will terminate after — (A) one (B) two (C) three (D) four decimal places.

Solution

Factorise the denominator: $1250 = 2 \times 5^4$. Since it contains only 2 and 5, the decimal terminates. The number of places equals the higher power, which is 4 (from 5^4). Check: multiply by $2^3/2^3$ to make the denominator $10^4 = 10000$, giving 11.6696 — four places.

Correct Answer: (D) four decimal places

Part C — Short Answer Questions with Reasoning

Worked Sample

> **QS1.** The values of the remainder r, when a positive integer a is divided by 3, are 0 and 1 only. Justify.

Solution

This statement is false. By Euclid's lemma, dividing by 3 gives $a = 3q + r$ with $0 \le r < 3$. So r can be 0, 1 OR 2 — three possible remainders, not two.

Exercise 1.2

> **Q1.** Can every positive integer be of the form $4q + 2$, where q is an integer? Justify.

Solution

No. Numbers of the form $4q + 2$ are always even (they leave remainder 2 on division by 4), so this form can never produce odd numbers like 1, 3 or 5. Hence not every positive integer fits this form.

> **Q2.** 'The product of two consecutive positive integers is divisible by 2.' True or false? Give reasons.

Solution

True. Out of any two back-to-back numbers, one must be even. An even number times anything is even, so the product always has 2 as a factor. Example: $4 \times 5 = 20$, which is divisible by 2.

> **Q3.** 'The product of three consecutive positive integers is divisible by 6.' True or false? Justify.

Solution

True. Among any three consecutive integers, at least one is even (giving a factor of 2) and exactly one is a multiple of 3 (giving a factor of 3). Since $2 \times 3 = 6$, the product is always divisible by 6. Example: $3 \times 4 \times 5 = 60$, divisible by 6.

> **Q4.** Can the square of any positive integer be of the form $3m + 2$, where m is a natural number? Justify.

Solution

No. Every integer is of the form $3k$, $3k + 1$ or $3k + 2$. Squaring each gives: $(3k)^2 = 9k^2 = 3m$; $(3k+1)^2 = 9k^2 + 6k + 1 = 3m + 1$; $(3k+2)^2 = 9k^2 + 12k + 4 = 3m + 1$. So a perfect square always leaves remainder 0 or 1 when divided by 3 — never 2.

> **Q5.** A positive integer is of the form $3q + 1$. Can its square be anything other than $3m + 1$ (i.e. $3m$ or $3m + 2$)? Justify.

Solution

No. Squaring $3q + 1$ gives $9q^2 + 6q + 1 = 3(3q^2 + 2q) + 1 = 3m + 1$. The square always lands in the $3m + 1$ form, so it can never be $3m$ or $3m + 2$.

> **Q6.** The numbers 525 and 3000 are both divisible only by 3, 5, 15, 25 and 75. What is HCF (525, 3000)? Justify.

Solution

The HCF is the LARGEST number that divides both. From the common divisors listed (3, 5, 15, 25, 75), the biggest is 75. Therefore HCF (525, 3000) = 75.

Q7. Explain why $3 \times 5 \times 7 + 7$ is a composite number.

Solution

Take 7 common from both terms: $3 \times 5 \times 7 + 7 = 7 \times (3 \times 5 + 1) = 7 \times 16 = 112$. Because the expression has factors other than 1 and itself (such as 7 and 16), it is composite, not prime.

> **Exam Tip**
>
> Whenever a sum can be factored by pulling out a common number, it is automatically composite. Spotting the common factor is the whole trick.

Q8. Can two numbers have 18 as their HCF and 380 as their LCM? Give reasons.

Solution

No. The HCF of two numbers must always divide their LCM exactly. Here $380 \div 18$ is not a whole number (18 does not divide 380), so such a pair of numbers cannot exist.

> **Important Note**
>
> A favourite exam trap: always test whether HCF divides LCM. If it does not, the situation is impossible.

Q9. Without long division, decide whether 987 / 10500 has a terminating or non-terminating decimal expansion. Give reasons.

Solution

First reduce the fraction. Both 987 and 10500 are divisible by 21: $987 \div 21 = 47$ and $10500 \div 21 = 500$. So the fraction becomes 47/500. Now $500 = 2^2 \times 5^3$, which contains only the primes 2 and 5. Therefore the decimal terminates.

Q10. A rational number's decimal expansion is 327.7081. What can you say about the prime factors of q when it is written as p/q?

Solution

Since the decimal stops after four places, it is a terminating decimal. By the rule for terminating decimals, the denominator q (in lowest terms) can only have the primes 2 and 5 — that is, $q = 2^m \times 5^n$. Here the number equals 3277081/10000, and $10000 = 2^4 \times 5^4$, confirming only the primes 2 and 5.

Part D — Short Answer Questions

Worked Samples

> **QS1.** Using Euclid's algorithm, check whether these pairs are co-prime: (i) 231, 396 (ii) 847, 2160.

Solution

(i) Find HCF of 396 and 231:

$$396 = 231 \times 1 + 165$$
$$231 = 165 \times 1 + 66$$
$$165 = 66 \times 2 + 33$$
$$66 = 33 \times 2 + 0$$

The HCF is 33. Since the HCF is not 1, the numbers are NOT co-prime.

(ii) Find HCF of 2160 and 847:

$$2160 = 847 \times 2 + 466$$
$$847 = 466 \times 1 + 381$$
$$466 = 381 \times 1 + 85$$
$$381 = 85 \times 4 + 41$$
$$85 = 41 \times 2 + 3$$
$$41 = 3 \times 13 + 2$$
$$3 = 2 \times 1 + 1$$
$$2 = 1 \times 2 + 0$$

The HCF is 1, so these two numbers ARE co-prime.

> **Concept Recall**
> Two numbers are co-prime when their only common factor is 1, i.e. their HCF equals 1.

> **QS2.** Show that the square of an odd positive integer is of the form 8m + 1, for some whole number m.

Solution

Any odd positive integer can be written as 2q + 1. Squaring:

$$(2q + 1)^2 = 4q^2 + 4q + 1 = 4q(q + 1) + 1$$

Now q(q + 1) is a product of two consecutive integers, so it is always even — write it as 2m. Substituting:

$$(2q + 1)^2 = 4 \times 2m + 1 = 8m + 1$$

This proves every odd square has the form 8m + 1.

QS3. Prove that √2 + √3 is irrational.

Solution

Assume the opposite — suppose √2 + √3 is rational and equals some rational number a. Then √2 = a − √3. Squaring both sides:

$$2 = a^2 - 2a\sqrt{3} + 3$$

Rearranging to isolate the root:

$$\sqrt{3} = (a^2 + 1) / (2a)$$

The right side is built only from rational numbers, so it is rational. But √3 is known to be irrational — a contradiction. Hence our assumption was wrong, and √2 + √3 must be irrational.

> **Exam Tip**
> Every 'prove irrational' proof follows the same recipe: assume it IS rational, do some algebra, and reach a statement that forces a known irrational (like √3) to be rational. That clash finishes the proof.

Exercise 1.3

Q1. Show that the square of any positive integer is of the form 4q or 4q + 1 for some integer q.

Solution

Every integer is either even (2k) or odd (2k + 1).

$$\text{Even: } (2k)^2 = 4k^2 = 4q$$

$$\text{Odd: } (2k + 1)^2 = 4k^2 + 4k + 1 = 4(k^2 + k) + 1 = 4q + 1$$

So a perfect square is always of the form 4q or 4q + 1 — never 4q + 2 or 4q + 3.

Q2. Show that the cube of any positive integer is of the form 4m, 4m + 1 or 4m + 3.

Solution

Write the integer as 4k, 4k + 1, 4k + 2 or 4k + 3 and cube each (keeping only the remainder on division by 4):

$$(4k)^3 = 64k^3 = 4m$$

$$(4k + 1)^3 = ... + 1 = 4m + 1$$

$$(4k + 2)^3 = 8(...) = 4m$$

$$(4k + 3)^3 = ... + 27 = 4m + 3 \quad (\text{since } 27 = 4\times6 + 3)$$

Collecting the possibilities, the cube is always of the form 4m, 4m + 1 or 4m + 3.

Q3. Show that the square of a positive integer cannot be of the form 5q + 2 or 5q + 3.

Solution

Any integer is 5k, 5k + 1, 5k + 2, 5k + 3 or 5k + 4. Squaring and reducing modulo 5 gives remainders 0, 1, 4, 4, 1 respectively. The only possible remainders are 0, 1 and 4 — so a square can never leave remainder 2 or 3, i.e. it is never of the form 5q + 2 or 5q + 3.

Q4. Show that the square of a positive integer cannot be of the form 6m + 2 or 6m + 5.

Solution

Write the integer as 6k + r with r = 0,1,2,3,4,5 and square it. The remainders on dividing the squares by 6 turn out to be 0, 1, 4, 3, 4, 1. The list of achievable remainders is {0, 1, 3, 4}. Since 2 and 5 never appear, a perfect square cannot be 6m + 2 or 6m + 5.

Q5. Show that the square of any odd integer is of the form 4q + 1.

Solution

An odd integer is 2k + 1. Squaring: $(2k + 1)^2 = 4k^2 + 4k + 1 = 4(k^2 + k) + 1 = 4q + 1$. Done.

Q6. If n is an odd integer, show that $n^2 - 1$ is divisible by 8.

Solution

Factor: $n^2 - 1 = (n - 1)(n + 1)$. For odd n, both n − 1 and n + 1 are even and consecutive even numbers. One of any two consecutive even numbers is a multiple of 4, so the product carries factors 4 and 2 together — that is 8. Hence 8 divides $n^2 - 1$.

Q7. Prove that if x and y are both odd positive integers, then $x^2 + y^2$ is even but not divisible by 4.

Solution

Let x = 2a + 1 and y = 2b + 1. Then:

$$\mathrm{x^2 + y^2 = (2a{+}1)^2 + (2b{+}1)^2 = 4a^2 + 4a + 1 + 4b^2 + 4b + 1}$$

$$\mathrm{= 4(a^2 + a + b^2 + b) + 2}$$

This is clearly even (it ends in + 2). But because of that leftover '+ 2', it leaves remainder 2 when divided by 4, so it is NOT divisible by 4.

Q8. Use Euclid's algorithm to find the HCF of 441, 567, 693.

Solution

Find the HCF of the first two, then combine with the third.

$$567 = 441 \times 1 + 126$$

$$441 = 126 \times 3 + 63$$

$$126 = 63 \times 2 + 0 \quad \rightarrow \quad \mathrm{HCF}(441,\ 567) = 63$$

Now take the HCF of 63 and 693:

$$693 = 63 \times 11 + 0 \quad \rightarrow \quad \mathrm{HCF} = 63$$

Therefore HCF (441, 567, 693) = 63.

Q9. Find the largest number that divides 1251, 9377 and 15628 leaving remainders 1, 2 and 3 respectively.

Solution

Subtract each remainder first, so the number divides these exactly: 1251 − 1 = 1250, 9377 − 2 = 9375, 15628 − 3 = 15625. The answer is HCF of 1250, 9375 and 15625.

$$9375 = 1250 \times 7 + 625$$

$$1250 = 625 \times 2 + 0 \quad \rightarrow \quad \mathrm{HCF}(1250,\, 9375) = 625$$

$$15625 = 625 \times 25 + 0 \quad \rightarrow \quad \mathrm{HCF} = 625$$

So the largest such number is 625.

Q10. Prove that √3 + √5 is irrational.

Solution

Assume √3 + √5 is rational, say equal to a. Then √3 = a − √5. Squaring:

$$3 = \mathrm{a}^2 - 2\mathrm{a}\sqrt{5} + 5$$

$$\sqrt{5} = (\mathrm{a}^2 + 2) \;/\; (2\mathrm{a})$$

The right side is rational, which would force √5 to be rational. But √5 is irrational — contradiction. So √3 + √5 is irrational.

Q11. Show that 12^n cannot end with the digit 0 or 5 for any natural number n.

Solution

A number ends in 0 or 5 only if 5 is one of its prime factors. But $12^n = (2^2 \times 3)^n = 2^{(2n)} \times 3^n$, which contains only the primes 2 and 3 — never 5. By the uniqueness of prime factorisation, 12^n can therefore never end in 0 or 5.

Q12. Three people on a morning walk have step lengths 40 cm, 42 cm and 45 cm. What is the minimum distance each should walk so that all cover the same distance in complete steps?

Solution

We need the smallest distance that is a whole-number multiple of each step length — that is the LCM of 40, 42 and 45.

$$40 = 2^3 \times 5$$

$$42 = 2 \times 3 \times 7$$

$$45 = 3^2 \times 5$$

Take the highest power of each prime: $2^3 \times 3^2 \times 5 \times 7 = 8 \times 9 \times 5 \times 7 = 2520$. So the minimum distance is 2520 cm (25.2 metres).

> **Exam Tip**
> 'Same distance / together again' problems are always LCM. 'Largest size that divides everything / greatest number' problems are always HCF.

Q13. Write the denominator of 257 / 5000 in the form $2^m \times 5^n$, then write its decimal expansion without dividing.

Solution

Factorise $5000 = 5^3 \times 2^3 \times$ ... let us check: $5000 = 8 \times 625 = 2^3 \times 5^4$. So m = 3 and n = 4. To convert to a power of ten denominator, the higher power is 4, so multiply top and bottom by 2^1:

$$257 \;/\; 5000 = (257 \times 2) \;/\; (5000 \times 2) = 514 \;/\; 10000 = 0.0514$$

The decimal terminates after four places: 0.0514.

Q14. Prove that $\sqrt{p} + \sqrt{q}$ is irrational, where p and q are primes.

Solution

Suppose $\sqrt{p} + \sqrt{q}$ is rational, call it a. Then $\sqrt{p} = a - \sqrt{q}$. Square both sides:

$$p = a^2 - 2a\sqrt{q} + q$$

$$\sqrt{q} = (a^2 + q - p)\ /\ (2a)$$

The right side is made entirely of rationals, so it would be rational. But $\sqrt{q}$ (the square root of a prime) is irrational — a contradiction. Hence $\sqrt{p} + \sqrt{q}$ is irrational.

Part E — Long Answer Questions

Worked Sample

QS1. Show that the square of an odd positive integer can be of the form 6q + 1 or 6q + 3.

Solution

Any positive integer is one of 6m, 6m+1, 6m+2, 6m+3, 6m+4, 6m+5. The odd ones among these are 6m + 1, 6m + 3 and 6m + 5. Square each:

$$(6m + 1)^2 = 36m^2 + 12m + 1 = 6(6m^2 + 2m) + 1 = 6q + 1$$

$$(6m + 3)^2 = 36m^2 + 36m + 9 = 6(6m^2 + 6m + 1) + 3 = 6q + 3$$

$$(6m + 5)^2 = 36m^2 + 60m + 25 = 6(6m^2 + 10m + 4) + 1 = 6q + 1$$

All three results fall into the form 6q + 1 or 6q + 3, which proves the statement.

Exercise 1.4

Q1. Show that the cube of a positive integer of the form 6q + r (r = 0,1,2,3,4,5) is also of the form 6m + r.

Solution

The idea: when you cube any number written as 6q + r and expand it, every term that contains the factor 6 collects into a single multiple of 6, and what is left over matches r^3 reduced modulo 6. Working each case, the remainder of the cube on division by 6 turns out to equal the original r:

$$r{=}0 \to \text{cube} \equiv 0;\quad r{=}1 \to 1;\quad r{=}2 \to 8 \equiv 2;\quad r{=}3 \to 27 \equiv 3;\quad r{=}4 \to 64 \equiv 4;\quad r{=}5 \to 125 \equiv 5$$

In every case the leftover equals r, so the cube is again of the form 6m + r.

Q2. Prove that one and only one out of n, n + 2 and n + 4 is divisible by 3, for any positive integer n.

Solution

By Euclid's lemma, n is of the form 3q, 3q + 1 or 3q + 2.

- If n = 3q: then n is divisible by 3. The others, n+2 = 3q+2 and n+4 = 3q+4 (=3(q+1)+1), are not.
- If n = 3q + 1: then n+2 = 3q+3 = 3(q+1) is divisible by 3, while n and n+4 are not.
- If n = 3q + 2: then n+4 = 3q+6 = 3(q+2) is divisible by 3, while n and n+2 are not.

In each case exactly one of the three is divisible by 3 — never zero, never two. This proves the claim.

Q3. Prove that one of any three consecutive positive integers must be divisible by 3.

Solution

Take three consecutive integers n, n+1, n+2. Writing n as 3q, 3q+1 or 3q+2 shows that within these three numbers the remainders on dividing by 3 cover all of 0, 1, 2 exactly once. Since one of those remainders is 0, one of the three numbers is a multiple of 3.

Q4. For any positive integer n, prove that $n^3 - n$ is divisible by 6.

Solution

Factor it: $n^3 - n = n(n^2 - 1) = (n - 1)\cdot n\cdot(n + 1)$. This is a product of three consecutive integers. Among any three consecutive integers, one is divisible by 3 (giving a factor of 3) and at least one is even (giving a factor of 2). So the product is divisible by $2 \times 3 = 6$.

> **Important Note**
>
> The trick 'product of k consecutive integers is divisible by k!' is worth memorising. Three consecutive integers → divisible by $3! = 6$; four consecutive → divisible by $4! = 24$, and so on.

Q5. Show that one and only one out of n, $n + 4$, $n + 8$, $n + 12$ and $n + 16$ is divisible by 5, for any positive integer n.

Solution

Write n as $5q + r$, where r is one of 0, 1, 2, 3, 4. Notice the five numbers add multiples of 4 — but more usefully, modulo 5 they read as r, $r+4$, $r+8$, $r+12$, $r+16$, which reduce to r, $r+4$, $r+3$, $r+2$, $r+1 \pmod 5$. These five remainders are exactly 0, 1, 2, 3, 4 in some order. Since 0 appears once and only once, precisely one of the five numbers is divisible by 5.

> **Exam Tip**
>
> Whenever you see a set of numbers in arithmetic progression and a divisor d, reduce each term modulo d. If the remainders cover $0\ldots d-1$ exactly once, then exactly one term is divisible by d.

Quick Revision Sheet (Night Before the Exam)

- Euclid's Lemma: $a = bq + r$ with $0 \le r < b$. Repeat to get HCF; last non-zero divisor is the HCF.
- Fundamental Theorem: prime factorisation is unique.
- HCF × LCM = product of the two numbers.
- HCF = product of lowest powers of common primes; LCM = product of highest powers of all primes.
- Terminating decimal ⇔ denominator (in lowest terms) = $2^m \times 5^n$; number of places = max(m, n).
- To prove a number irrational: assume rational, isolate the root, reach a contradiction.
- Product of k consecutive integers is divisible by k! (so 3 consecutive → 6, etc.).
- 'Together again / same length' = LCM. 'Greatest number that divides' = HCF.
- HCF must always divide the LCM — a quick validity check for HCF/LCM problems.