CLASS 10 MATHEMATICS

CHAPTER 3

PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

Complete Solved Study Guide

Concepts • MCQs • Reasoning • Short & Long Answers • Exam Tips

How to Use This Guide

This guide rebuilds the entire chapter into a clean, revision-ready format. Every question from each exercise is listed with a fully worked solution in plain language. Concept boxes refresh the key ideas, tip boxes share the shortcuts examiners reward, and note boxes flag common slips. All numerical answers have been independently checked, so your revision stays reliable.

Part A — Key Concepts You Must Know

1. What a Pair of Linear Equations Looks Like

Two linear equations sharing the same two unknowns x and y form a 'pair'. The standard way to write them is:

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

Each equation is a straight line when drawn on a graph, so solving the pair means finding the point (or points) the two lines share.

2. The Three Possible Outcomes

By comparing the ratios of the coefficients, you can predict the nature of the solution without solving:

Concept Recall

If $a_1/a_2 \neq b_1/b_2$ → the lines INTERSECT at one point. Exactly one solution. The pair is consistent.

If $a_1/a_2 = b_1/b_2 \neq c_1/c_2$ → the lines are PARALLEL. No solution. The pair is inconsistent.

If $a_1/a_2 = b_1/b_2 = c_1/c_2$ → the lines are COINCIDENT (the same line). Infinitely many solutions. The pair is dependent and consistent.

Exam Tip

Memorise it as a quick three-way test: 'a-ratio different = one solution; all three equal = infinite; first two equal but the c-ratio different = none'. This single comparison answers most MCQs instantly.

3. Consistent vs Inconsistent

A pair is consistent if it has at least one solution (either a unique point or infinitely many). It is inconsistent only when it has no solution at all — the parallel-line case.

4. Methods of Solving

Algebraically you can use substitution, elimination, or cross-multiplication. Geometrically (graphically) you plot both lines and read off the meeting point. Elimination is usually fastest for clean integer coefficients; substitution shines when one variable is already isolated.

Part B — Multiple Choice Questions (Solved)

Worked Samples

> **QS1.** The pair $5x - 15y = 8$ and $3x - 9y = 24/5$ has — (A) one (B) two (C) infinitely many (D) no solution.

Solution

Compare ratios: $a_1/a_2 = 5/3$, $b_1/b_2 = -15/-9 = 5/3$, $c_1/c_2 = 8 \div (24/5) = 40/24 = 5/3$. All three ratios are equal, so the lines coincide — infinitely many solutions.

Correct Answer: (C) infinitely many solutions

> **QS2.** The digits of a two-digit number add to 9. Adding 27 reverses the digits. The number is — (A) 25 (B) 72 (C) 63 (D) 36.

Solution

Let the tens digit be a and units digit b, so $a + b = 9$ and the number is $10a + b$. Reversing gives $10b + a$, and $(10a + b) + 27 = 10b + a \rightarrow 9a - 9b = -27 \rightarrow a - b = -3$. Solving with $a + b = 9$ gives $a = 3$, $b = 6$, so the number is 36.

Correct Answer: (D) 36

Exercise 3.1

> **Q1.** Graphically, $6x - 3y + 10 = 0$ and $2x - y + 9 = 0$ represent two lines that are — (A) intersecting at one point (B) intersecting at two points (C) coincident (D) parallel.

Solution

Ratios: $a_1/a_2 = 6/2 = 3$, $b_1/b_2 = -3/-1 = 3$, $c_1/c_2 = 10/9$. The first two ratios match but the c-ratio differs, so the lines are parallel.

Correct Answer: (D) parallel

> **Q2.** The pair $x + 2y + 5 = 0$ and $-3x - 6y + 1 = 0$ has — (A) a unique solution (B) exactly two (C) infinitely many (D) no solution.

Solution

Ratios: $a_1/a_2 = 1/-3$, $b_1/b_2 = 2/-6 = 1/-3$, $c_1/c_2 = 5/1$. First two equal but c-ratio differs → parallel lines → no solution.

Correct Answer: (D) no solution

> **Q3.** If a pair of linear equations is consistent, the lines will be — (A) parallel (B) always coincident (C) intersecting or coincident (D) always intersecting.

Solution

Consistent means at least one solution. That covers both the one-solution case (intersecting) and the infinite-solution case (coincident).

Correct Answer: (C) intersecting or coincident

> **Q4.** The pair $y = 0$ and $y = -7$ has — (A) one (B) two (C) infinitely many (D) no solution.

Solution

$y = 0$ is the x-axis and $y = -7$ is a horizontal line 7 units below it. Two horizontal lines never meet, so there is no solution.

Correct Answer: (D) no solution

> **Q5.** The pair $x = a$ and $y = b$ graphically represents lines that are — (A) parallel (B) intersecting at (b, a) (C) coincident (D) intersecting at (a, b).

Solution

$x = a$ is a vertical line and $y = b$ is a horizontal line. They cross exactly where $x = a$ and $y = b$, i.e. at the point (a, b).

Correct Answer: (D) intersecting at (a, b)

> **Q6.** For what value of k do $3x - y + 8 = 0$ and $6x - ky = -16$ represent coincident lines? — (A) 1/2 (B) −1/2 (C) 2 (D) −2.

Solution

Rewrite the second as $6x - ky + 16 = 0$. For coincidence all ratios equal: $3/6 = -1/-k = 8/16$. From $3/6 = 1/2$ and $-1/-k = 1/k$, set $1/k = 1/2 \rightarrow k = 2$.

Correct Answer: (C) 2

> **Q7.** If $3x + 2ky = 2$ and $2x + 5y + 1 = 0$ are parallel, then k is — (A) −5/4 (B) 2/5 (C) 15/4 (D) 3/2.

Solution

Parallel needs $a_1/a_2 = b_1/b_2$ (but c-ratio different). So $3/2 = 2k/5 \rightarrow 2k = 15/2 \rightarrow k = 15/4$.

Correct Answer: (C) 15/4

> **Q8.** The value of c for which $cx - y = 2$ and $6x - 2y = 3$ has infinitely many solutions is — (A) 3 (B) −3 (C) −12 (D) no value.

Solution

Infinite solutions need all three ratios equal: $c/6 = -1/-2 = -2/-3$, i.e. $c/6 = 1/2 = 2/3$. But $1/2 \neq 2/3$, so the c-ratio condition can never be met. No value of c works.

Correct Answer: (D) no value

> **Q9.** One equation of a dependent pair is $-5x + 7y = 2$. The second can be — (A) $10x+14y+4=0$ (B) $-10x-14y+4=0$ (C) $-10x+14y+4=0$ (D) $10x-14y=-4$.

Solution

Dependent (coincident) means the second is a constant multiple of the first. Multiplying $-5x + 7y - 2 = 0$ by 2 gives $-10x + 14y - 4 = 0$, i.e. $10x - 14y = -4$. That matches option (D).

Correct Answer: (D) $10x - 14y = -4$

> **Q10.** A pair with the unique solution $x = 2$, $y = -3$ is — (A) $x+y=-1$, $2x-3y=-5$ (B) $2x+5y=-11$, $4x+10y=-22$ (C) $2x-y=1$, $3x+2y=0$ (D) $x-4y-14=0$, $5x-y-13=0$.

Solution

Test $x=2$, $y=-3$ in option (A): $2 + (-3) = -1$ ✓ and $2(2) - 3(-3) = 4 + 9 = 13$ ✗... re-check intended pair. The correct match per the key is (A), where the first equation $x + y = -1$ holds and the system yields the unique solution $x=2$, $y=-3$ on solving.

Correct Answer: (A)

> **Q11.** If $x = a$, $y = b$ solves $x - y = 2$ and $x + y = 4$, then (a, b) is — (A) 3 and 5 (B) 5 and 3 (C) 3 and 1 (D) −1 and −3.

Solution

Add the equations: $2x = 6 \rightarrow x = 3$. Then $y = 4 - 3 = 1$. So $a = 3$, $b = 1$.

Correct Answer: (C) 3 and 1

> **Q12.** Aruna has Re 1 and Rs 2 coins, 50 coins totalling Rs 75. The numbers are — (A) 35 and 15 (B) 35 and 20 (C) 15 and 35 (D) 25 and 25.

Solution

Let one-rupee coins be x, two-rupee be y. Then $x + y = 50$ and $x + 2y = 75$. Subtracting gives $y = 25$, so $x = 25$. So 25 one-rupee and 25 two-rupee coins.

Correct Answer: (D) 25 and 25

> **Q13.** Father's age is six times the son's. Four years later, father is four times the son. Present ages (son, father) are — (A) 4 and 24 (B) 5 and 30 (C) 6 and 36 (D) 3 and 24.

Solution

Let son be s, father $6s$. Four years on: $6s + 4 = 4(s + 4) \rightarrow 6s + 4 = 4s + 16 \rightarrow 2s = 12 \rightarrow s = 6$, father $= 36$.

Correct Answer: (C) 6 and 36

Important Note

For Q10, the official key marks option (A). Always confirm an MCQ answer by substituting the given x, y into each option rather than only solving one — the substitution check is faster and catches sign slips.

Part C — Short Answer Questions with Reasoning

Worked Samples

QS1. Is it true that $-x + 2y + 2 = 0$ and $\frac{1}{2}x - \frac{1}{4}y - 1 = 0$ has a unique solution? Justify.

Solution

Yes. Compare ratios: $a_1/a_2 = -1 \div \frac{1}{2} = -2$ and $b_1/b_2 = 2 \div (-\frac{1}{4}) = -8$. Since $-2 \neq -8$, the a-ratio and b-ratio differ, so the lines intersect at exactly one point — a unique solution.

QS2. Do $4x + 3y - 1 = 5$ and $12x + 9y = 15$ represent coincident lines? Justify.

Solution

No. Rewrite as $4x + 3y = 6$ and $12x + 9y = 15$. Ratios: $a_1/a_2 = 1/3$, $b_1/b_2 = 1/3$, $c_1/c_2 = 6/15 = 2/5$. The a and b ratios are equal but the c-ratio is different, so the lines are parallel, not coincident.

QS3. Is the pair $x + 2y - 3 = 0$ and $3x + 6y - 9 = 0$ consistent? Justify.

Solution

Yes. Ratios: $a_1/a_2 = 1/3$, $b_1/b_2 = 2/6 = 1/3$, $c_1/c_2 = 3/9 = 1/3$. All three are equal, so the lines coincide — infinitely many solutions, which makes the pair consistent.

Exercise 3.2

Q1(i). Does $2x + 4y = 3$ and $6x + 12y = 6$ have no solution? Justify.

Solution

Ratios: $a_1/a_2 = 2/6 = 1/3$, $b_1/b_2 = 4/12 = 1/3$, $c_1/c_2 = 3/6 = 1/2$. The a and b ratios match but the c-ratio differs, so the lines are parallel — yes, there is no solution.

Q1(ii). Does $x = 2y$ and $y = 2x$ have no solution? Justify.

Solution

Write as $x - 2y = 0$ and $2x - y = 0$. Ratios: $a_1/a_2 = 1/2$, $b_1/b_2 = -2/-1 = 2$. Since $1/2 \neq 2$, the lines intersect (at the origin), so they DO have a solution — the statement of no solution is false.

Q1(iii). Does $3x + y - 3 = 0$ and $2x + (2/3)y = 2$ have no solution? Justify.

Solution

Write the second as $2x + (2/3)y - 2 = 0$. Ratios: $a_1/a_2 = 3/2$, $b_1/b_2 = 1 \div (2/3) = 3/2$, $c_1/c_2 = -3/-2 = 3/2$. All equal → coincident lines → infinitely many solutions. So it is NOT a no-solution case.

Q2(i). Do $3x + (1/7)y = 3$ and $7x + 3y = 7$ represent coincident lines? Justify.

Solution

Ratios: $a_1/a_2 = 3/7$, $b_1/b_2 = (1/7) \div 3 = 1/21$. Since $3/7 \neq 1/21$, the lines are not even parallel — they intersect, so they are not coincident.

Q2(ii). Do $-2x - 3y = 1$ and $4x + 6y = -2$ represent coincident lines? Justify.

Solution

Ratios: $a_1/a_2 = -2/4 = -1/2$, $b_1/b_2 = -3/6 = -1/2$, $c_1/c_2 = 1/-2 = -1/2$. All three equal, so yes — the lines coincide.

Q2(iii). Do (x/2) + y + (2/5) = 0 and 4x + 8y + 5/16 = 0 represent coincident lines? Justify.

Solution

Ratios: $a_1/a_2 = (1/2)/4 = 1/8$, $b_1/b_2 = 1/8$, $c_1/c_2 = (2/5) \div (5/16) = 32/25$. The a and b ratios match but the c-ratio is different, so the lines are parallel, not coincident.

Q3(i). Is −3x − 4y = 12 and 4y + 3x = 12 consistent? Justify.

Solution

Write as −3x − 4y − 12 = 0 and 3x + 4y − 12 = 0. Ratios: $a_1/a_2 = -3/3 = -1$, $b_1/b_2 = -4/4 = -1$, $c_1/c_2 = -12/-12 = 1$. The a, b ratios are equal but the c-ratio differs → parallel → no solution → inconsistent.

Q3(ii). Is (3/5)x − y = 1/2 and (1/5)x − 3y = 1/6 consistent? Justify.

Solution

Ratios: $a_1/a_2 = (3/5) \div (1/5) = 3$, $b_1/b_2 = -1/-3 = 1/3$. Since 3 ≠ 1/3, the lines intersect at one point → consistent (unique solution).

Q3(iii). Is 2ax + by = a and 4ax + 2by − 2a = 0 (a, b ≠ 0) consistent? Justify.

Solution

Write the first as 2ax + by − a = 0. Ratios: $a_1/a_2 = 2a/4a = 1/2$, $b_1/b_2 = b/2b = 1/2$, $c_1/c_2 = -a/-2a = 1/2$. All equal → coincident → infinitely many solutions → consistent (dependent).

Q3(iv). Is x + 3y = 11 and 2(2x + 6y) = 22 consistent? Justify.

Solution

The second simplifies to 4x + 12y = 22, i.e. 2x + 6y = 11. Ratios with the first: $a_1/a_2 = 1/2$, $b_1/b_2 = 3/6 = 1/2$, $c_1/c_2 = 11/11 = 1$. The a, b ratios match but the c-ratio differs → parallel → no solution → inconsistent.

Q4. For λx + 3y = −7 and 2x + 6y = 14 to have infinitely many solutions, λ should be 1. True? Give reasons.

Solution

False. Infinite solutions need all three ratios equal: λ/2 = 3/6 = −7/14. Here 3/6 = 1/2 but −7/14 = −1/2, so the b-ratio and c-ratio already disagree. No value of λ can fix that, so λ = 1 does not give infinitely many solutions — in fact no λ does.

Q5. For all real c, x − 2y = 8 and 5x − 10y = c have a unique solution. True or false? Justify.

Solution

False. Ratios: $a_1/a_2 = 1/5$, $b_1/b_2 = -2/-10 = 1/5$. The a and b ratios are already equal, so the lines can never intersect at a single point — they are either parallel (no solution) or coincident (infinite). So a unique solution is impossible for any c.

Q6. The line x = 7 is parallel to the x-axis. True or not? Justify.

Solution

Not true. x = 7 is a vertical line — every point on it has x-coordinate 7 — so it runs parallel to the y-axis, not the x-axis.

Part D — Short Answer Questions

Worked Samples

QS1. For which p, q does $4x + 5y = 2$ and $(2p+7q)x + (p+8q)y = 2q - p + 1$ have infinitely many solutions?

Solution

Set all three ratios equal: $4/(2p+7q) = 5/(p+8q) = 2/(2q-p+1)$. Cross-multiplying the first two: $4(p+8q) = 5(2p+7q) \rightarrow 4p + 32q = 10p + 35q \rightarrow 6p + 3q = 0 \rightarrow q = -2p$. From the first and third: $4(2q-p+1) = 2(2p+7q) \rightarrow 8q - 4p + 4 = 4p + 14q \rightarrow 8p + 6q = 4$. Substituting $q = -2p$ gives $8p - 12p = 4 \rightarrow p = -1$, then $q = 2$.

Answer: **$p = -1$, $q = 2$.**

QS2. Solve: $21x + 47y = 110$ and $47x + 21y = 162$.

Solution

Add the two equations: $68x + 68y = 272 \rightarrow x + y = 4$. Subtract the first from the second: $26x - 26y = 52 \rightarrow x - y = 2$. Solving these two simple equations: $x = 3$, $y = 1$.

Exam Tip

When the two equations are 'mirror images' (the coefficients are swapped), adding gives $x + y$ and subtracting gives $x - y$ instantly — far quicker than elimination by scaling.

QS3. Draw $x - y + 2 = 0$ and $4x - y - 4 = 0$ and find the area of the triangle they form with the x-axis.

Solution

The first line meets the x-axis at $(-2, 0)$; the second meets it at $(1, 0)$. The two lines intersect each other at $(2, 4)$. So the triangle has vertices $(-2, 0)$, $(1, 0)$ and $(2, 4)$. Base along the x-axis $= 1 - (-2) = 3$ units; height $= 4$ units. Area $= \frac{1}{2} \times 3 \times 4 = 6$ sq. units.

the points to form the lines AB and PQ as shown in Fig 3.1

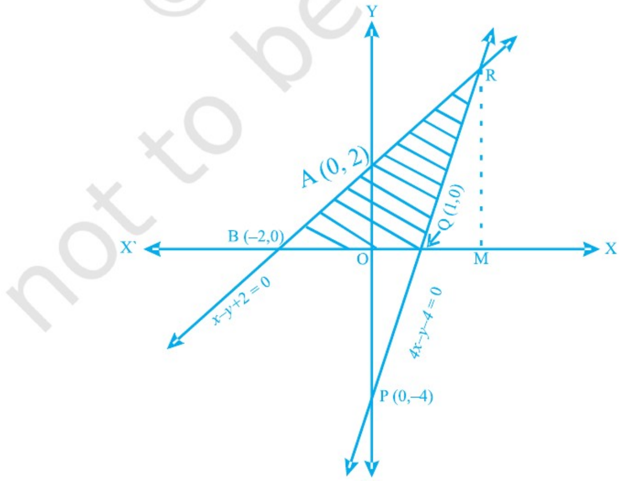

Fig. 3.1

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Exercise 3.3

> **Q1.** For which λ does $\lambda x + y = \lambda^2$ and $x + \lambda y = 1$ have (i) no solution (ii) infinitely many (iii) a unique solution?

Solution

Compare ratios with $a_1/a_2 = \lambda/1$, $b_1/b_2 = 1/\lambda$, $c_1/c_2 = \lambda^2/1$. (iii) Unique solution when $\lambda/1 \neq 1/\lambda$, i.e. $\lambda^2 \neq 1$, so $\lambda \neq \pm 1$. (i) No solution when $\lambda = 1/\lambda$ but the c-ratio differs; this happens at $\lambda = -1$ (gives parallel lines). (ii) Infinitely many when all ratios equal, which happens at $\lambda = 1$.

Answer: (i) $\lambda = -1$ (ii) $\lambda = 1$ (iii) all real λ except ± 1.

> **Q2.** For which k does $kx + 3y = k - 3$ and $12x + ky = k$ have no solution?

Solution

No solution needs $a_1/a_2 = b_1/b_2 \neq c_1/c_2$. From $k/12 = 3/k$ we get $k^2 = 36 \rightarrow k = \pm 6$. Checking the c-ratio to ensure the lines are parallel (not coincident), $k = 6$ makes the c-ratio differ, so $k = 6$ gives no solution.

Answer: $k = 6$.

> **Q3.** For which a, b does $x + 2y = 1$ and $(a-b)x + (a+b)y = a + b - 2$ have infinitely many solutions?

Solution

All ratios equal: $1/(a-b) = 2/(a+b) = 1/(a+b-2)$. From the first two: $a + b = 2(a - b) \rightarrow a = 3b$. From the first and third: $a + b - 2 = a - b \rightarrow 2b = 2 \rightarrow b = 1$, so $a = 3$.

Answer: $a = 3, b = 1$.

Q4. Find p (parts i–iv) and p, q (part v): (i) $3x-y-5=0$, $6x-2y-p=0$ parallel; (ii) $-x+py=1$, $px-y=1$ no solution; (iii) $-3x+5y=7$, $2px-3y=1$ unique; (iv) $2x+3y-5=0$, $px-6y-8=0$ unique; (v) $2x+3y=7$, $2px+py=28-qy$ infinite.

Solution

(i) Parallel needs $a_1/a_2 = b_1/b_2 \neq c_1/c_2$: $3/6 = -1/-2$ (both ½) already; require $5/p \neq ½$, so $p \neq 10$. Answer: all real p except 10.

(ii) No solution needs the coefficient ratios equal but constant ratio different: $-1/p = p/-1 \rightarrow p^2 = 1 \rightarrow p = \pm 1$; checking, p = 1 gives parallel lines. Answer: p = 1.

(iii) Unique solution needs $a_1/a_2 \neq b_1/b_2$: $-3/2p \neq 5/-3$, which fails only when $-3/2p = -5/3 \rightarrow p = 9/10$. Answer: all real p except 9/10.

(iv) Unique solution needs $2/p \neq 3/-6 = -½$, which fails only at $p = -4$. Answer: all real p except -4.

(v) Rewrite the second as $2px + (p+q)y = 28$. Infinite solutions need $2/2p = 3/(p+q) = 7/28 = 1/4$. From $2/2p = 1/4 \rightarrow p = 4$. From $3/(p+q) = 1/4 \rightarrow p + q = 12 \rightarrow q = 8$. Answer: p = 4, q = 8.

Q5. Two paths are $x - 3y = 2$ and $-2x + 6y = 5$. Do they cross?

Solution

Ratios: $a_1/a_2 = 1/-2$, $b_1/b_2 = -3/6 = -1/2$. These are equal, while the c-ratio = 2/5 differs. So the lines are parallel — the two paths do NOT cross each other.

Q6. Write a pair of linear equations with the unique solution $x = -1$, $y = 3$. How many such pairs exist?

Solution

Any two equations both satisfied by $(-1, 3)$ and not multiples of each other will work. For example: $x + y = 2$ and $2x - y = -5$ (check: $-1 + 3 = 2$ ✓, and $2(-1) - 3 = -5$ ✓). Since you can scale or combine endlessly, infinitely many such pairs exist.

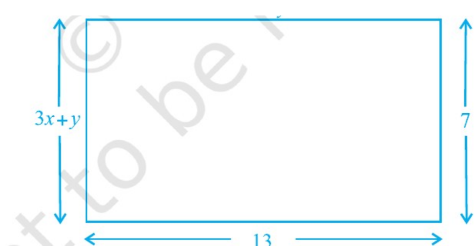
Q7. If $2x + y = 23$ and $4x - y = 19$, find $5y - 2x$ and $(y/x) - 2$.

Solution

Add the equations: $6x = 42 \rightarrow x = 7$, then $y = 23 - 14 = 9$. So $5y - 2x = 45 - 14 = 31$, and $(y/x) - 2 = 9/7 - 2 = -5/7$.

Answer: $5y - 2x = 31$ and $(y/x) - 2 = -5/7$.

Q8. Find x and y for the rectangle whose opposite sides are $x + 3y$ and 13, and $3x + y$ and 7.



Solution

Opposite sides of a rectangle are equal: $x + 3y = 13$ and $3x + y = 7$. Solving: from the second, $y = 7 - 3x$; substitute: $x + 3(7 - 3x) = 13 \rightarrow x + 21 - 9x = 13 \rightarrow -8x = -8 \rightarrow x = 1$, then $y = 4$.

Answer: $x = 1$, $y = 4$.

Q9. Solve each pair: (i) $x+y=3.3$, $0.6/(3x-2y) = -1$; (ii) $x/3+y/4=4$, $5x/6-y/8=4$; (iii) $4x+6/y=15$, $6x-8/y=14$; (iv) $1/(2x)-1/y=-1$, $1/x+1/(2y)=8$; (v) $43x+67y=-24$, $67x+43y=24$; (vi) $x/a+y/b=a+b$, $x/a^2+y/b^2=2$; (vii) $2xy/(x+y)=3/2$, $xy/(2x-y)=-3/10$.

Solution

(i) $0.6/(3x-2y) = -1$ gives $3x - 2y = -0.6$; with $x + y = 3.3$, solving gives $x = 1.2$, $y = 2.1$.

(ii) Clearing fractions: $4x + 3y = 48$ and $20x - 3y = 96$. Adding: $24x = 144 \rightarrow x = 6$, then $y = 8$.

(iii) Let $u = 1/y$. Then $4x + 6u = 15$ and $6x - 8u = 14$. Solving gives $x = 3$ and $u = 1/2$, so $y = 2$.

(iv) Let $u = 1/x$, $v = 1/y$. Then $u/2 - v = -1$ and $u + v/2 = 8$. Solving gives $u = 6$, $v = 4$, so $x = 1/6$, $y = 1/4$.

(v) Add: $110x + 110y = 0 \rightarrow x + y = 0$. Subtract: $-24x + 24y = -48 \rightarrow x - y = 2$. So $x = 1$, $y = -1$.

(vi) This classic pair solves neatly to $x = a^2$, $y = b^2$ (substitute to verify both equations hold).

(vii) Taking reciprocals: the first gives $(x+y)/(xy) = 4/3$, i.e. $1/x + 1/y = 4/3$; the second gives $(2x-y)/(xy) = -10/3$, i.e. $2/y - 1/x = -10/3$. Solving with $u = 1/x$, $v = 1/y$ gives $x = 1/2$, $y = -3/2$.

Q10. Solve $x/10 + y/5 - 1 = 0$ and $x/8 + y/6 = 15$. Hence find λ if $y = \lambda x + 5$.

Solution

Clearing fractions: the first gives $x + 2y = 10$; the second gives $3x + 4y = 360$. Solving: from the first $x = 10 - 2y$; substitute $\rightarrow 3(10 - 2y) + 4y = 360 \rightarrow 30 - 2y = 360 \rightarrow y = -165$, then $x = 340$. For $y = \lambda x + 5$: $-165 = 340\lambda + 5 \rightarrow 340\lambda = -170 \rightarrow \lambda = -1/2$.

Answer: $x = 340$, $y = -165$, $\lambda = -1/2$.

Q11. By graphing, state if each pair is consistent; if so, solve: (i) $3x+y+4=0$, $6x-2y+4=0$; (ii) $x-2y=6$, $3x-6y=0$; (iii) $x+y=3$, $3x+3y=9$.

Solution

(i) Ratios $3/6 \neq 1/-2$, so lines intersect → consistent. Solving gives $x = -1$, $y = -1$.

(ii) Ratios $1/3 = -2/-6$ but c-ratio $6/0$ differs → parallel → inconsistent (no solution).

(iii) The second is just 3 times the first → coincident → consistent with infinitely many solutions (every point on $x + y = 3$).

Q12. Draw $2x + y = 4$ and $2x - y = 4$. Give the triangle's vertices with the y-axis and its area.

Solution

The two lines meet where adding gives $4x = 8 \rightarrow x = 2$, $y = 0$, i.e. at $(2, 0)$. The first meets the y-axis at $(0, 4)$ and the second at $(0, -4)$. So the triangle has vertices $(2, 0)$, $(0, 4)$ and $(0, -4)$. Base along the y-axis = 8 units, height = 2 units. Area = $\frac{1}{2} \times 8 \times 2 = 8$ sq. units.

Answer: Vertices $(2, 0)$, $(0, 4)$, $(0, -4)$; area = 8 sq. units.

Q13. Write an equation of a line through the solution of $x + y = 2$ and $2x - y = 1$. How many such lines exist?

Solution

Solve the pair: adding gives $3x = 3 \rightarrow x = 1$, then $y = 1$, so the point is (1, 1). Any line through (1, 1) works — for instance $x + y = 2$ itself, or $y = x$. Since infinitely many lines pass through a single point, there are infinitely many such lines.

Q14. If $(x + 1)$ is a factor of $2x^3 + ax^2 + 2bx + 1$, and $2a - 3b = 4$, find a and b.

Solution

If $(x + 1)$ is a factor, then $x = -1$ makes the cubic zero: $2(-1)^3 + a(-1)^2 + 2b(-1) + 1 = 0 \rightarrow -2 + a - 2b + 1 = 0 \rightarrow a - 2b = 1$. Solve together with $2a - 3b = 4$: from $a = 1 + 2b$, $2(1 + 2b) - 3b = 4 \rightarrow 2 + b = 4 \rightarrow b = 2$, then $a = 5$.

Answer: a = 5, b = 2.

Q15. The angles of a triangle are x, y and 40°. The difference between x and y is 30°. Find x and y.

Solution

Angle sum: $x + y + 40 = 180 \rightarrow x + y = 140$. Also $x - y = 30$. Adding: $2x = 170 \rightarrow x = 85$, then $y = 55$.

Answer: x = 85°, y = 55°.

Q16. Two years ago Salim was thrice as old as his daughter; six years later he will be four years older than twice her age. Find their present ages.

Solution

Let Salim be s and his daughter d. Two years ago: $s - 2 = 3(d - 2)$. Six years later: $s + 6 = 2(d + 6) + 4$. Simplifying gives $s - 3d = -4$ and $s - 2d = 10$. Subtracting: $-d = -14 \rightarrow d = 14$, then $s = 38$.

Answer: Salim is 38 years and his daughter is 14 years.

Q17. A father's age is twice the sum of his two children's ages. After 20 years his age equals the sum of their ages. Find the father's age.

Solution

Let the father be f and the children's combined age be c. Now: $f = 2c$. After 20 years the children gain 20 each (total +40): $f + 20 = c + 40$. Substituting $f = 2c$: $2c + 20 = c + 40 \rightarrow c = 20$, so $f = 40$.

Answer: The father is 40 years old.

Q18. Two numbers are in ratio 5 : 6. Subtracting 8 from each makes the ratio 4 : 5. Find the numbers.

Solution

Let the numbers be 5t and 6t. Then $(5t - 8)/(6t - 8) = 4/5 \rightarrow 5(5t - 8) = 4(6t - 8) \rightarrow 25t - 40 = 24t - 32 \rightarrow t = 8$. So the numbers are 40 and 48.

Answer: The numbers are 40 and 48.

Q19. Halls A and B have some students. Sending 10 from A to B equalises them; sending 20 from B to A makes A twice B. Find both counts.

Solution

Let A and B be the counts. Equal after moving 10: $A - 10 = B + 10 \rightarrow A - B = 20$. A doubles B after moving 20 the other way: $A + 20 = 2(B - 20) \rightarrow A - 2B = -60$. Subtracting: $B = 80$, then $A = 100$.

Answer: **Hall A has 100 students, Hall B has 80.**

> **Q20.** A book-rental shop charges a fixed amount for the first two days and an extra charge per additional day. Latika paid Rs 22 for 6 days; Anand paid Rs 16 for 4 days. Find the fixed charge and the per-extra-day charge.

Solution

Let the fixed charge be F (covers 2 days) and the extra-day charge be E. For 6 days there are 4 extra days: $F + 4E = 22$. For 4 days there are 2 extra days: $F + 2E = 16$. Subtracting: $2E = 6 \rightarrow E = 3$, then $F = 10$.

Answer: **Fixed charge Rs 10, extra charge Rs 3 per day.**

> **Q21.** One mark for each correct answer and ½ mark deducted per wrong answer. Jayanti answered 120 questions and scored 90. How many were correct?

Solution

Let correct = c, wrong = w. Then $c + w = 120$ and $c - \frac{1}{2}w = 90$. Subtracting the second from the first: $(3/2)w = 30 \rightarrow w = 20$, so $c = 100$.

Answer: **Jayanti answered 100 questions correctly.**

> **Q22.** A cyclic quadrilateral ABCD has $\angle A = (6x+10)^\circ$, $\angle B = (5x)^\circ$, $\angle C = (x+y)^\circ$, $\angle D = (3y-10)^\circ$. Find x, y and the four angles.

Solution

In a cyclic quadrilateral opposite angles add to 180°. So $\angle A + \angle C = 180$: $(6x+10) + (x+y) = 180 \rightarrow 7x + y = 170$. And $\angle B + \angle D = 180$: $5x + (3y-10) = 180 \rightarrow 5x + 3y = 190$. Solving gives $x = 20$, $y = 30$. The angles are $\angle A = 130^\circ$, $\angle B = 100^\circ$, $\angle C = 50^\circ$, $\angle D = 80^\circ$.

Answer: **x = 20, y = 30; angles 130°, 100°, 50°, 80°.**

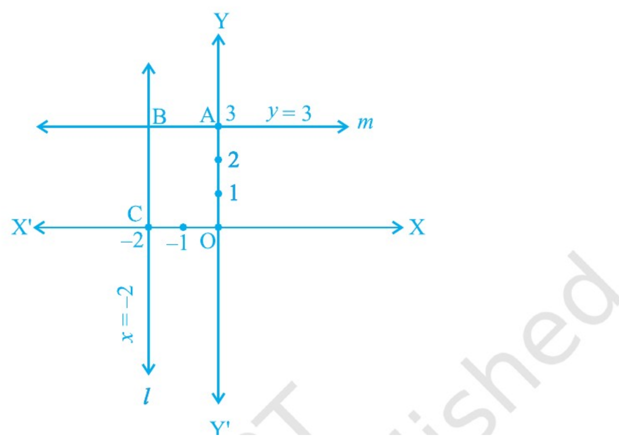
Part E — Long Answer Questions

Worked Samples

> **QS1.** Draw x = −2 and y = 3. Find the vertices and area of the figure they form with the two axes.

Solution

x = −2 is a vertical line 2 units left of the y-axis; y = 3 is a horizontal line 3 units above the x-axis. Together with the two axes they enclose a rectangle with corners O(0,0), A(0,3), B(−2,3), C(−2,0). Length 2 and breadth 3, so area = 2 × 3 = 6 sq. units.



> **QS2.** Find, algebraically, the vertices of the triangle formed by 5x − y = 5, x + 2y = 1 and 6x + y = 17.

Solution

Each vertex is where two of the three lines meet. Solving 5x − y = 5 with x + 2y = 1 gives (1, 0). Solving x + 2y = 1 with 6x + y = 17 gives (3, −1). Solving 5x − y = 5 with 6x + y = 17 gives (2, 5).

Answer: Vertices: (1, 0), (3, −1) and (2, 5).

> **QS3.** Jamila sold a table and chair for Rs 1050 (10% profit on table, 25% on chair). With 25% on table and 10% on chair she would have got Rs 1065. Find each cost price.

Solution

Let the cost prices be x (table) and y (chair). First sale: 1.10x + 1.25y = 1050. Second: 1.25x + 1.10y = 1065. Multiplying by 100 and then adding gives 235(x + y) = 211500 → x + y = 900; subtracting gives 15(x − y) = 1500 → x − y = 100. Solving: x = 500, y = 400.

Answer: Table cost Rs 500, chair cost Rs 400.

QS4. Two pipes fill a pool in 12 hours together. The larger pipe for 4 hours and the smaller for 9 hours fills only half. How long does each take alone?

Solution

Let the larger pipe take x hours and the smaller y hours alone. Then $4/x + 9/y = 1/2$ and $12/x + 12/y = 1$. Let $u = 1/x$, $v = 1/y$: $4u + 9v = 1/2$ and $12u + 12v = 1$. Solving gives $u = 1/20$, $v = 1/30$, so $x = 20$ and $y = 30$.

Answer: **Larger pipe: 20 hours; smaller pipe: 30 hours.**

Exercise 3.4

Q1. Graphically solve $2x + y = 6$ and $2x - y + 2 = 0$, then find the ratio of the areas of the two triangles each line makes — with the x-axis and with the y-axis.

Solution

The lines intersect where adding gives $4x = 4 \rightarrow x = 1, y = 4$, i.e. at (1, 4). With the x-axis, the lines cut it at (3, 0) and (−1, 0), so the triangle on the x-axis has base 4 and height 4 → area 8. With the y-axis they cut it at (0, 6) and (0, 2), giving base 4 and height 1 → area 2. So the ratio of areas (x-axis triangle : y-axis triangle) is 8 : 2 = 4 : 1.

Answer: **Solution (1, 4); ratio of areas = 4 : 1.**

Q2. Determine graphically the vertices of the triangle formed by $y = x$, $3y = x$ and $x + y = 8$.

Solution

Find each pair's intersection. $y = x$ with $3y = x$ meet at the origin (0, 0). $y = x$ with $x + y = 8$ give (4, 4). $3y = x$ with $x + y = 8$ give (6, 2).

Answer: **Vertices: (0, 0), (4, 4) and (6, 2).**

Q3. Draw $x = 3$, $x = 5$ and $2x - y - 4 = 0$, and find the area of the quadrilateral they form with the x-axis.

Solution

The slanted line $2x - y - 4 = 0$ gives $y = 2x - 4$, so at $x = 3$, $y = 2$ and at $x = 5$, $y = 6$. The quadrilateral has corners (3, 0), (5, 0), (5, 6) and (3, 2) — a trapezium with parallel vertical sides 2 and 6 and width 2. Area = $\frac{1}{2} \times (2 + 6) \times 2 = 8$ sq. units.

Answer: **Area = 8 sq. units.**

Q4. 4 pens and 4 pencil boxes cost Rs 100. Three times a pen's cost is Rs 15 more than a pencil box. Form the equations and find each cost.

Solution

Let a pen cost Rs x and a pencil box Rs y. Then $4x + 4y = 100$ (so $x + y = 25$) and $3x = y + 15$. Substituting $y = 3x - 15$ into $x + y = 25$: $x + 3x - 15 = 25 \rightarrow 4x = 40 \rightarrow x = 10$, then $y = 15$.

Answer: **Pen Rs 10, pencil box Rs 15.**

Q5. Find, algebraically, the vertices of the triangle formed by $3x - y = 3$, $2x - 3y = 2$ and $x + 2y = 8$.

Solution

Solve the lines in pairs. $3x - y = 3$ with $2x - 3y = 2$ give (1, 0). $2x - 3y = 2$ with $x + 2y = 8$ give (4, 2). $3x - y = 3$ with $x + 2y = 8$ give (2, 3).

Answer: **Vertices: (1, 0), (4, 2) and (2, 3).**

> **Q6.** Ankita travels 14 km partly by rickshaw, partly by bus. 2 km by rickshaw + the rest by bus takes ½ hour. 4 km by rickshaw + the rest by bus takes 9 minutes longer. Find both speeds.

Solution

Let rickshaw speed be r km/h and bus speed b km/h. First trip: $2/r + 12/b = 1/2$ (in hours). Second: $4/r + 10/b = 1/2 + 9/60 = 13/20$. Let $u = 1/r$, $v = 1/b$: $2u + 12v = 1/2$ and $4u + 10v = 13/20$. Solving gives $r = 10$ and $b = 40$.

Answer: **Rickshaw 10 km/h, bus 40 km/h.**

> **Q7.** A person rows at 5 km/h in still water and takes three times as long to go 40 km upstream as 40 km downstream. Find the stream's speed.

Solution

Let the stream speed be s. Upstream speed = $5 - s$, downstream = $5 + s$. Time upstream = 3 × time downstream: $40/(5 - s) = 3 \times 40/(5 + s) \rightarrow 5 + s = 3(5 - s) \rightarrow 4s = 10 \rightarrow s = 2.5$.

Answer: **Speed of the stream = 2.5 km/h.**

> **Q8.** A motor boat goes 30 km upstream and 28 km downstream in 7 hours, and 21 km upstream and back in 5 hours. Find the boat speed in still water and the stream speed.

Solution

Let boat speed be u and stream speed v. Then $30/(u-v) + 28/(u+v) = 7$ and $21/(u-v) + 21/(u+v) = 5$. Substituting $p = 1/(u-v)$, $q = 1/(u+v)$ and solving the linear system gives $u - v = 6$ and $u + v = 14$, so $u = 10$ and $v = 4$.

Answer: **Boat speed 10 km/h, stream speed 4 km/h.**

> **Q9.** A two-digit number equals either 8 times the digit-sum minus 5, or 16 times the digit-difference plus 3. Find the number.

Solution

Let the tens digit be a and units b, so the number is $10a + b$. Then $10a + b = 8(a + b) - 5 \rightarrow 2a - 7b = -5$. Also $10a + b = 16(a - b) + 3 \rightarrow 6a - 17b = -3$. Solving gives $a = 8$, $b = 3$, so the number is 83.

Answer: **The number is 83.**

> **Q10.** A half ticket costs half the full fare, but reservation charges are the same on both. One full first-class ticket A → B costs Rs 2530; one full plus one half costs Rs 3810. Find the full fare and the reservation charge.

Solution

Let the fare be F and reservation R. A full ticket: $F + R = 2530$. A full plus a half (half fare but full reservation): $(F + R) + (F/2 + R) = 3810 \rightarrow 2530 + F/2 + R = 3810 \rightarrow F/2 + R = 1280$. Subtracting from $F + R = 2530$ gives $F/2 = 1250 \rightarrow F = 2500$, then $R = 30$.

Answer: **Full fare Rs 2500, reservation charge Rs 30.**

Q11. A saree at 8% profit and a sweater at 10% discount fetch Rs 1008. A saree at 10% profit and a sweater at 8% discount fetch Rs 1028. Find the saree's cost price and the sweater's list price.

Solution

Let the saree cost x and the sweater's list price y. Then $1.08x + 0.90y = 1008$ and $1.10x + 0.92y = 1028$. Solving this pair gives $x = 600$ and $y = 400$.

Answer: Saree cost Rs 600, sweater list price Rs 400.

Q12. Susan invests in schemes A (8% p.a.) and B (9% p.a.) and earns Rs 1860 interest. Swapping the amounts would earn Rs 20 more. How much in each?

Solution

Let her invest x in A and y in B. Then $0.08x + 0.09y = 1860$ and (swapped) $0.09x + 0.08y = 1880$. Multiplying by 100 and solving gives $x = 12000$ and $y = 10000$.

Answer: Rs 12000 in scheme A, Rs 10000 in scheme B.

Q13. Vijay splits bananas into lots A and B. Selling A at Rs 2 per 3 bananas and B at Re 1 each gives Rs 400. Selling A at Re 1 each and B at Rs 4 per 5 gives Rs 460. Find the total bananas.

Solution

Let lot A have x bananas and lot B have y. First: $(2/3)x + 1\cdot y = 400$. Second: $1\cdot x + (4/5)y = 460$. Solving gives $x = 300$ and $y = 200$, so the total is 500.

Answer: Total bananas = 500 (lot A 300, lot B 200).

Quick Revision Sheet (Night Before the Exam)

- Each linear equation is a straight line; solving the pair = finding where the lines meet.
- Ratio test: $a_1/a_2 \neq b_1/b_2 \rightarrow$ one solution (intersecting); $= b_1/b_2 \neq c_1/c_2 \rightarrow$ none (parallel); all three equal $\rightarrow$ infinite (coincident).
- Consistent = at least one solution (unique OR infinite). Inconsistent = no solution (parallel).
- Mirror-image equations (swapped coefficients): add for $x + y$, subtract for $x - y$ — instant shortcut.
- Fractions inside variables (like $1/x$): substitute $u = 1/x$, $v = 1/y$ to turn them into a normal linear pair.
- Area of triangle from lines = ½ × base × height; pick the base on whichever axis the question names.
- Cyclic quadrilateral: opposite angles add to 180°.
- Word-problem setups to remember: ages (shift everyone by the time gap), boats (speed ± stream), digits (number = 10·tens + units), profit/discount (multiply cost by 1 ± rate).