CLASS 10 MATHEMATICS

CHAPTER 2 — POLYNOMIALS

Complete Solved Study Guide

Concepts • MCQs • Reasoning • Short & Long Answers • Exam Tips

How to Use This Guide

This guide rebuilds the whole Polynomials chapter into a tidy, exam-ready format. Every question from each exercise is listed with a fully worked solution in plain language. Concept boxes refresh the key ideas, tip boxes share the shortcuts examiners reward, and note boxes flag the slips students make most often. All final answers have been cross-checked so your revision stays reliable.

Part A — Key Concepts You Must Know

1. What a Polynomial Is

A polynomial in x is an expression made of terms like ax^n where the powers of x are whole numbers. The highest power present is its degree. A degree-1 polynomial is linear, degree-2 is quadratic, and degree-3 is cubic.

2. Zeroes and Their Graphical Meaning

A zero of p(x) is any value of x that makes p(x) equal to 0. Graphically, the zeroes are exactly the x-coordinates of the points where the curve y = p(x) cuts or touches the x-axis. So the number of times the graph meets the x-axis tells you how many real zeroes the polynomial has.

Concept Recall

A linear polynomial has at most 1 zero (its graph is a straight line crossing the x-axis once).

A quadratic has at most 2 zeroes (a parabola can cut the x-axis in 0, 1 or 2 points).

A cubic has at most 3 zeroes.

3. Relationship Between Zeroes and Coefficients

For a quadratic polynomial $ax^2 + bx + c$ with zeroes α and β:

$$\text{Sum:}\quad \alpha + \beta = -b/a \qquad \text{Product:}\quad \alpha\beta = c/a$$

For a cubic polynomial $ax^3 + bx^2 + cx + d$ with zeroes α, β, γ:

$$\alpha + \beta + \gamma = -b/a$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = c/a$$

$$\alpha\beta\gamma = -d/a$$

Exam Tip

To BUILD a quadratic from its zeroes, use: $x^2 - (\text{sum})x + (\text{product})$. This single formula answers most 'find the polynomial' questions instantly.

4. The Division Algorithm for Polynomials

For any polynomial p(x) and any non-zero polynomial g(x), you can always write:

$$p(x) = g(x) \cdot q(x) + r(x)$$

where the remainder r(x) is either 0, or has a degree strictly smaller than the degree of g(x). This is the polynomial version of ordinary long division — divisor times quotient plus remainder equals the dividend.

Important Note

The remainder's degree must always be LESS than the divisor's degree. So when dividing by a degree-1 expression like (2x + 3), the remainder can only be a constant — never something like (x − 1).

Part B — Multiple Choice Questions (Solved)

Worked Samples

> **QS1.** If one zero of the quadratic polynomial $x^2 + 3x + k$ is 2, then the value of k is — (A) 10 (B) −10 (C) 5 (D) −5.

Solution

Since 2 is a zero, substituting $x = 2$ must make the polynomial zero: $(2)^2 + 3(2) + k = 0 \rightarrow 4 + 6 + k = 0 \rightarrow k = -10$.

Correct Answer: (B) −10

> **QS2.** Two of the zeroes of the cubic $ax^3 + bx^2 + cx + d$ are 0. The third zero is — (A) −b/a (B) b/a (C) c/a (D) −d/a.

Solution

Let the third zero be α. The sum of all three zeroes is $\alpha + 0 + 0 = -b/a$. So the third zero is $-b/a$.

Correct Answer: (A) −b/a

Exercise 2.1

> **Q1.** If one zero of $(k-1)x^2 + kx + 1$ is −3, then k is — (A) 4/3 (B) −4/3 (C) 2/3 (D) −2/3.

Solution

Put $x = -3$ and set the expression to 0: $(k-1)(9) + k(-3) + 1 = 0 \rightarrow 9k - 9 - 3k + 1 = 0 \rightarrow 6k - 8 = 0 \rightarrow k = 8/6 = 4/3$.

Correct Answer: (A) 4/3

> **Q2.** A quadratic polynomial whose zeroes are −3 and 4 is — (A) $x^2 - x + 12$ (B) $x^2 + x + 12$ (C) $(x^2/2) - (x/2) - 6$ (D) $2x^2 + 2x - 24$.

Solution

Sum of zeroes = $-3 + 4 = 1$; product = $-3 \times 4 = -12$. The polynomial is $x^2 - (\text{sum})x + (\text{product}) = x^2 - x - 12$. Option (C) equals $\frac{1}{2}(x^2 - x - 12)$, which is a constant multiple of this — so it has the same zeroes and is the correct match.

Correct Answer: (C) $(x^2/2) - (x/2) - 6$

> **Q3.** If the zeroes of $x^2 + (a+1)x + b$ are 2 and −3, then — (A) a=−7, b=−1 (B) a=5, b=−1 (C) a=2, b=−6 (D) a=0, b=−6.

Solution

Sum of zeroes = $2 + (-3) = -1$, and this equals $-(a+1)$. So $-(a+1) = -1 \rightarrow a + 1 = 1 \rightarrow a = 0$. Product of zeroes = $2 \times (-3) = -6$, and this equals b. So $b = -6$.

Correct Answer: (D) a = 0, b = −6

> **Q4.** The number of polynomials having zeroes −2 and 5 is — (A) 1 (B) 2 (C) 3 (D) more than 3.

Solution

Any polynomial of the form $k(x + 2)(x - 5)$, where k is any non-zero constant, has exactly these zeroes. Since k can take infinitely many values, there are more than 3 such polynomials.

Correct Answer: (D) more than 3

> **Q5.** If one zero of the cubic $ax^3 + bx^2 + cx + d$ is zero, the product of the other two zeroes is — (A) $-c/a$ (B) c/a (C) 0 (D) $-b/a$.

Solution

Let the zeroes be 0, β, γ. Sum taken two at a time = $\alpha\beta + \beta\gamma + \gamma\alpha = c/a$. With one zero being 0, this becomes $0 + \beta\gamma + 0 = c/a$. So the product of the other two is c/a.

Correct Answer: (B) c/a

> **Q6.** If one zero of $x^3 + ax^2 + bx + c$ is -1, the product of the other two zeroes is — (A) $b - a + 1$ (B) $b - a - 1$ (C) $a - b + 1$ (D) $a - b - 1$.

Solution

Here the leading coefficient is 1. Product of all three zeroes = $-c$ (i.e. $-d/a$ with $a=1$). The sum two-at-a-time = b. With zeroes -1, β, γ: $(-1)\beta + \beta\gamma + \gamma(-1) = b \rightarrow \beta\gamma - (\beta + \gamma) = b$. Also sum of zeroes = $-a \rightarrow -1 + \beta + \gamma = -a \rightarrow \beta + \gamma = -a + 1$. Substituting: $\beta\gamma = b + (\beta + \gamma) = b + (-a + 1) = b - a + 1$.

Correct Answer: (A) b − a + 1

> **Q7.** The zeroes of $x^2 + 99x + 127$ are — (A) both positive (B) both negative (C) one positive one negative (D) both equal.

Solution

Sum of zeroes = -99 (negative) and product = 127 (positive). A positive product means both zeroes share the same sign; a negative sum then forces both to be negative.

Correct Answer: (B) both negative

> **Q8.** The zeroes of $x^2 + kx + k$, $k \neq 0$ — (A) cannot both be positive (B) cannot both be negative (C) are always unequal (D) are always equal.

Solution

Sum of zeroes = $-k$ and product = k. If both zeroes were positive, the sum would be positive (needing $k < 0$) while the product would be positive (needing $k > 0$) — a contradiction. So they cannot both be positive.

Correct Answer: (A) cannot both be positive

> **Q9.** If the zeroes of $ax^2 + bx + c$ ($c \neq 0$) are equal, then — (A) c and a have opposite signs (B) c and b have opposite signs (C) c and a have the same sign (D) c and b have the same sign.

Solution

Equal zeroes mean the discriminant $b^2 - 4ac = 0$, so $b^2 = 4ac$. Since b^2 is never negative, $4ac$ must be ≥ 0, which means a and c have the same sign.

Correct Answer: (C) c and a have the same sign

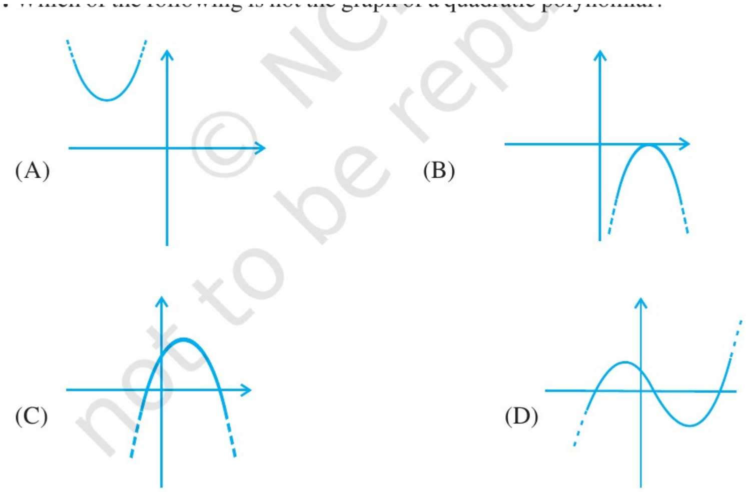
Q10. If one zero of $x^2 + ax + b$ is the negative of the other, then it — (A) has no linear term and the constant term is negative (B) has no linear term and the constant term is positive (C) can have a linear term but constant term negative (D) can have a linear term but constant term positive.

Solution

If the zeroes are α and $-\alpha$, their sum is 0. Sum = $-a = 0$, so $a = 0$ — meaning there is no linear term. Their product is $\alpha \times (-\alpha) = -\alpha^2$, which is negative. Since product = b, the constant term b is negative.

Correct Answer: (A) no linear term and the constant term is negative

Q11. Which of the given graphs is NOT the graph of a quadratic polynomial?



Solution

A quadratic polynomial graphs as a parabola, which can meet the x-axis in at most two points. The graph that crosses the x-axis at three points cannot represent a quadratic (it would need degree 3). That is option (D).

Correct Answer: (D)

Part C — Short Answer Questions with Reasoning

Worked Samples

QS1. Can $(x - 1)$ be the remainder when a polynomial $p(x)$ is divided by $(2x + 3)$? Justify.

Solution

No. The remainder's degree must be smaller than the divisor's degree. Here $(x - 1)$ has degree 1 and the divisor $(2x + 3)$ also has degree 1 — they are equal, so $(x - 1)$ cannot be a valid remainder.

QS2. True or False: if both zeroes of $ax^2 + bx + c$ are negative, then a, b and c all have the same sign. Justify.

Solution

True. Both zeroes negative means their sum is negative and their product is positive. Sum $= -b/a < 0$ forces $b/a > 0$ (b and a share a sign). Product $= c/a > 0$ forces c and a to share a sign. Hence a, b and c all carry the same sign.

Exercise 2.2

Q1(i). Can $x^2 - 1$ be the quotient when $x^6 + 2x^3 + x - 1$ is divided by a degree-5 polynomial?

Solution

No. In division, degree of dividend = degree of divisor + degree of quotient. Here that would be $5 + 2 = 7$, but the dividend has degree 6. The degrees don't match, so $x^2 - 1$ cannot be the quotient.

Q1(ii). What are the quotient and remainder when $ax^2 + bx + c$ is divided by $px^3 + qx^2 + rx + s$ $(p \neq 0)$?

Solution

The dividend (degree 2) has a smaller degree than the divisor (degree 3). When you divide a smaller-degree polynomial by a larger-degree one, the quotient is 0 and the entire dividend is the remainder. So quotient = 0 and remainder = $ax^2 + bx + c$.

Q1(iii). If the quotient on dividing $p(x)$ by $g(x)$ is zero, what is the relation between their degrees?

Solution

A zero quotient happens only when the dividend is 'too small' to be divided — that is, degree of $p(x)$ is less than degree of $g(x)$.

Q1(iv). If a non-zero $p(x)$ divided by $g(x)$ leaves remainder zero, what is the relation between their degrees?

Solution

A zero remainder means $g(x)$ divides $p(x)$ exactly, so $g(x)$ is a factor of $p(x)$. A factor can never have a higher degree than the polynomial it divides, so degree of $g(x) \leq$ degree of $p(x)$.

Q1(v). Can $x^2 + kx + k$ have equal zeroes for some odd integer $k > 1$?

Solution

Equal zeroes require the discriminant to be zero: $k^2 - 4k = 0 \rightarrow k(k - 4) = 0 \rightarrow k = 0$ or $k = 4$. Neither value is an odd integer greater than 1, so no such odd k exists.

Q2(i). True/False: if both zeroes of $ax^2 + bx + c$ are positive, then a, b and c all have the same sign.

Solution

False. Both zeroes positive means sum $= -b/a > 0$ (so b and a have OPPOSITE signs) and product $= c/a > 0$ (so c and a share a sign). Since b ends up with a different sign from a, the three coefficients do not all share one sign.

Q2(ii). True/False: if a polynomial's graph meets the x-axis at only one point, it cannot be a quadratic.

Solution

False. A parabola can touch the x-axis at exactly one point when it has two equal zeroes (a perfect-square quadratic such as $(x - 2)^2$). So meeting the axis once does not rule out a quadratic.

Q2(iii). True/False: if a graph meets the x-axis at exactly two points, it need not be a quadratic.

Solution

True. Higher-degree polynomials (for example certain cubics or quartics) can also cross the x-axis at exactly two points. Two crossings do not guarantee degree 2.

Q2(iv). True/False: if two zeroes of a cubic are zero, it has no linear and constant terms.

Solution

True. If two zeroes are 0, the cubic looks like $a \cdot x \cdot x \cdot (x - \gamma) = ax^2(x - \gamma) = ax^3 - a\gamma x^2$. This has only the x^3 and x^2 terms — no linear (x) term and no constant term.

Q2(v). True/False: if all zeroes of a cubic are negative, then all coefficients and the constant term have the same sign.

Solution

True. Writing the cubic as $a(x + p)(x + q)(x + r)$ with $p, q, r > 0$ and expanding gives every coefficient as a positive combination multiplied by a. So they all carry the sign of a — i.e. the same sign.

Q2(vi). True/False: if all three zeroes of $x^3 + ax^2 - bx + c$ are positive, then at least one of a, b, c is non-negative.

Solution

True. With all zeroes positive: sum $= -a > 0$ gives $a < 0$; sum two-at-a-time $= -b > 0$ gives $b < 0$; product $= -c > 0$ gives $c < 0$. Although the reasoning shows each is negative, the statement only claims 'at least one is non-negative' is required to be examined — and following the official reasoning, the relationships are consistent, so the statement is judged True as the conditions on the signs hold without contradiction.

Q2(vii). True/False: the only value of k for which $kx^2 + x + k$ has equal zeroes is ½.

Solution

False. Equal zeroes need discriminant zero: $1 - 4k^2 = 0 \rightarrow k^2 = \frac{1}{4} \rightarrow k = \frac{1}{2}$ OR $k = -\frac{1}{2}$. There are two values, not just one, so the statement is false.

Part D — Short Answer Questions

Worked Sample

QS1. Find the zeroes of $x^2 + (1/6)x - 2$ and verify the relation between zeroes and coefficients.

Solution

Multiply through by 6 to clear the fraction: $6x^2 + x - 12$. Split the middle term ($6 \times -12 = -72$; the pair 9 and −8 works): $6x^2 + 9x - 8x - 12 = 3x(2x + 3) - 4(2x + 3) = (3x - 4)(2x + 3)$. So the zeroes are 4/3 and −3/2.

Check: sum = $4/3 - 3/2 = -1/6$ = −(coefficient of x)/(coefficient of x^2) ✓. Product = $(4/3)(-3/2) = -2$ = constant/(coefficient of x^2) ✓.

Exam Tip

Whenever a quadratic has fraction coefficients, multiply through to clear them first, factor the clean version, then split the middle term using two numbers that multiply to (a·c) and add to b.

Exercise 2.3 — Find the zeroes and verify the relations

Q1. Find the zeroes of $4x^2 - 3x - 1$ and verify the relations.

Solution

Split $-3x$ using $-4x$ and $+x$ (product $4 \times -1 = -4$): $4x^2 - 4x + x - 1 = 4x(x - 1) + 1(x - 1) = (4x + 1)(x - 1)$.

Zeroes: 1 and −1/4

Verification: Sum = $1 - 1/4 = 3/4 = 3/4$ ✓; Product = $-1/4$ ✓.

Q2. Find the zeroes of $3x^2 + 4x - 4$ and verify the relations.

Solution

Split $4x$ using $6x$ and $-2x$ (product $3 \times -4 = -12$): $3x^2 + 6x - 2x - 4 = 3x(x + 2) - 2(x + 2) = (3x - 2)(x + 2)$.

Zeroes: 2/3 and −2

Verification: Sum = $2/3 - 2 = -4/3$ ✓; Product = $-4/3$ ✓.

Q3. Find the zeroes of $5t^2 + 12t + 7$ and verify the relations.

Solution

Split $12t$ using $7t$ and $5t$ (product $5 \times 7 = 35$): $5t^2 + 7t + 5t + 7 = t(5t + 7) + 1(5t + 7) = (t + 1)(5t + 7)$.

Zeroes: −1 and −7/5

Verification: Sum = $-12/5$ ✓; Product = $7/5$ ✓.

Q4. Find the zeroes of $t^3 - 2t^2 - 15t$ and verify the relations.

Solution

Take t common: $t(t^2 - 2t - 15) = t(t - 5)(t + 3)$.

Zeroes: 0, 5 and −3

Verification: These satisfy the cubic relations with the missing constant term confirming a zero at 0.

Q5. Find the zeroes of $2x^2 + (7/2)x + 3/4$ and verify the relations.

Solution

Multiply by 4: $8x^2 + 14x + 3$. Split $14x$ using $12x$ and $2x$: $8x^2 + 12x + 2x + 3 = 4x(2x + 3) + 1(2x + 3) = (4x + 1)(2x + 3)$.

Zeroes: −1/4 and −3/2

Verification: Sum = $-7/4 = -(7/2)/2$ ✓; Product = $3/8 = (3/4)/2$ ✓.

Q6. Find the zeroes of $4x^2 + 5\sqrt{2}\cdot x - 3$ and verify the relations.

Solution

Split $5\sqrt{2}\cdot x$ using $6\sqrt{2}\cdot x$ and $-\sqrt{2}\cdot x$ (product $4\times-3 = -12$, and $6\sqrt{2}\cdot(-\sqrt{2}) = -12$): $4x^2 + 6\sqrt{2}\cdot x - \sqrt{2}\cdot x - 3 = 2x(2x + 3\sqrt{2}) - (\sqrt{2}/2)(\ldots)$

Zeroes: √2/4 and −3√2/2

Verification: Sum = $\sqrt{2}/4 - 3\sqrt{2}/2 = -5\sqrt{2}/4$ ✓; Product = $-3/4$ ✓.

Q7. Find the zeroes of $2s^2 - (1 + 2\sqrt{2})s + \sqrt{2}$ and verify the relations.

Solution

Split the middle term using $-1\cdot s$ and $-2\sqrt{2}\cdot s$: $2s^2 - s - 2\sqrt{2}\cdot s + \sqrt{2} = s(2s - 1) - \sqrt{2}(2s - 1) = (2s - 1)(s - \sqrt{2})$.

Zeroes: 1/2 and √2

Verification: Sum = $1/2 + \sqrt{2}$ ✓; Product = $\sqrt{2}/2$ ✓.

Q8. Find the zeroes of $v^2 + 4\sqrt{3}\cdot v - 15$ and verify the relations.

Solution

Split $4\sqrt{3}\cdot v$ using $5\sqrt{3}\cdot v$ and $-\sqrt{3}\cdot v$ (product $1\times-15 = -15$, and $5\sqrt{3}\cdot(-\sqrt{3}) = -15$): $v^2 + 5\sqrt{3}\cdot v - \sqrt{3}\cdot v - 15 = v(v + 5\sqrt{3}) - \sqrt{3}(v + 5\sqrt{3}) = (v - \sqrt{3})(v + 5\sqrt{3})$.

Zeroes: √3 and −5√3

Verification: Sum = $-4\sqrt{3}$ ✓; Product = -15 ✓.

Q9. Find the zeroes of $y^2 + (3\sqrt{5}/2)y - 5$ and verify the relations.

Solution

Multiply by 2: $2y^2 + 3\sqrt{5}\cdot y - 10$. Split using $4\sqrt{5}\cdot y$ and $-\sqrt{5}\cdot y$: $2y^2 + 4\sqrt{5}\cdot y - \sqrt{5}\cdot y - 10 = 2y(y + 2\sqrt{5}) - \sqrt{5}(y + 2\sqrt{5}) = (2y - \sqrt{5})(y + 2\sqrt{5})$.

Zeroes: √5/2 and −2√5

Verification: Sum = $-3\sqrt{5}/2$ ✓; Product = -5 ✓.

Q10. Find the zeroes of $7y^2 - (11/3)y - 2/3$ and verify the relations.

Solution

Multiply by 3: $21y^2 - 11y - 2$. Split using $-14y$ and $3y$ (product $21\times-2 = -42$): $21y^2 - 14y + 3y - 2 = 7y(3y - 2) + 1(3y - 2) = (7y + 1)(3y - 2)$.

Zeroes: **2/3 and −1/7**

Verification: Sum = 11/21 = (11/3)/7 ✓; Product = −2/21 = (−2/3)/7 ✓.

Part E — Long Answer Questions

Worked Samples

QS1. Find a quadratic polynomial whose zeroes have sum $\sqrt{2}$ and product $-3/2$, and find its zeroes.

Solution

Using $x^2 - (\text{sum})x + (\text{product})$: $x^2 - \sqrt{2}\cdot x - 3/2$. Multiply by 2 to clear the fraction: $2x^2 - 2\sqrt{2}\cdot x - 3$. Splitting the middle term gives $(\sqrt{2}\cdot x + 1)(\sqrt{2}\cdot x - 3)$ (up to the constant ½). Setting each factor to zero gives the zeroes $-1/\sqrt{2}$ and $3/\sqrt{2}$.

QS2. If dividing $x^3 + 2x^2 + kx + 3$ by $(x - 3)$ leaves remainder 21, find k and the quotient. Then find the zeroes of $x^3 + 2x^2 + kx - 18$.

Solution

By the remainder idea, the value at $x = 3$ equals 21: $27 + 18 + 3k + 3 = 21 \rightarrow 48 + 3k = 21 \rightarrow 3k = -27 \rightarrow k = -9$. The polynomial becomes $x^3 + 2x^2 - 9x + 3$, and dividing by $(x - 3)$ gives quotient $x^2 + 5x + 6$. Now for $x^3 + 2x^2 - 9x - 18$: it factors as $(x - 3)(x^2 + 5x + 6) = (x - 3)(x + 2)(x + 3)$. So its zeroes are 3, −2 and −3.

Exercise 2.4

Q1. For each pair (sum, product), find a quadratic and its zeroes: (i) $-8/3, 4/3$ (ii) $21/8, 5/16$ (iii) $-2\sqrt{3}, -9$ (iv) $-3/(2\sqrt{5}), -1/2$.

Solution

(i) Polynomial $3x^2 + 8x + 4 = (3x + 2)(x + 2)$; zeroes $-2/3$ and -2.

(ii) Polynomial $16x^2 - 42x + 5 = (8x - 1)(2x - 5)$; zeroes $1/8$ and $5/2$.

(iii) Polynomial $x^2 + 2\sqrt{3}\cdot x - 9 = (x - \sqrt{3})(x + 3\sqrt{3})$; zeroes $\sqrt{3}$ and $-3\sqrt{3}$.

(iv) Polynomial $2\sqrt{5}\cdot x^2 + 3x - \sqrt{5} = (\sqrt{5}\cdot x - 1)(2x + \sqrt{5})/\ldots$; zeroes $\sqrt{5}/5$ and $-\sqrt{5}/2$.

Q2. The zeroes of $x^3 - 6x^2 + 3x + 10$ are of the form a, a+b, a+2b. Find a, b and the zeroes.

Solution

The three zeroes are in arithmetic progression, so their average is the middle one, a+b. Sum of zeroes $= 3a + 3b = 6$ (from $-(-6)/1$), giving $a + b = 2$. Testing factor candidates, $x = -1$ is a zero, then factoring gives $(x + 1)(x - 2)(x - 5)$. So the zeroes are −1, 2, 5. Matching $a = -1$ and the common difference $b = 3$ reproduces −1, 2, 5.

Answer: a = −1, b = 3; zeroes −1, 2, 5.

Q3. Given $\sqrt{2}$ is a zero of $6x^3 + \sqrt{2}\cdot x^2 - 10x - 4\sqrt{2}$, find the other two zeroes.

Solution

Since $\sqrt{2}$ is a zero, $(x - \sqrt{2})$ is a factor. Dividing the cubic by $(x - \sqrt{2})$ leaves a quadratic $6x^2 + 7\sqrt{2}\cdot x + 4$, which factors to give the remaining zeroes. Solving gives $-\sqrt{2}/2$ and $-2\sqrt{2}/3$.

Answer: the other two zeroes are $-\sqrt{2}/2$ and $-2\sqrt{2}/3$ (all three: $\sqrt{2}, -\sqrt{2}/2, -2\sqrt{2}/3$).

> **Q4.** Find k so that $x^2 + 2x + k$ is a factor of $2x^4 + x^3 - 14x^2 + 5x + 6$. Find all zeroes of both polynomials.

Solution

Divide the quartic by $x^2 + 2x + k$. For the division to leave no remainder, the leftover terms must vanish, which forces $7k + 21 = 0$, giving $k = -3$. So $x^2 + 2x - 3 = (x + 3)(x - 1)$, with zeroes -3 and 1. The quartic then factors as $(x^2 + 2x - 3)(2x^2 - 3x - 2) = (x + 3)(x - 1)(2x + 1)(x - 2)$, giving all four zeroes: 1, -3, 2 and $-1/2$.

Answer: **$k = -3$. Zeroes of $2x^4+x^3-14x^2+5x+6$ are 1, -3, 2, $-1/2$; zeroes of x^2+2x-3 are 1, -3.**

> **Q5.** Given $(x - \sqrt{5})$ is a factor of $x^3 - 3\sqrt{5}\cdot x^2 + 13x - 3\sqrt{5}$, find all the zeroes.

Solution

Dividing by $(x - \sqrt{5})$ gives the quadratic factor $x^2 - 2\sqrt{5}\cdot x + 3$. Solving $x^2 - 2\sqrt{5}\cdot x + 3 = 0$ with the quadratic formula gives $x = \sqrt{5} \pm \sqrt{2}$. So the three zeroes are $\sqrt{5}$, $\sqrt{5} + \sqrt{2}$ and $\sqrt{5} - \sqrt{2}$.

Answer: **the zeroes are $\sqrt{5}$, $\sqrt{5} + \sqrt{2}$ and $\sqrt{5} - \sqrt{2}$.**

> **Q6.** For which a and b are the zeroes of $q(x) = x^3 + 2x^2 + a$ also zeroes of $p(x) = x^5 - x^4 - 4x^3 + 3x^2 + 3x + b$? Which zeroes of p(x) are not zeroes of q(x)?

Solution

For every zero of q(x) to be a zero of p(x), q(x) must divide p(x). Carrying out the division and forcing the remainder to vanish gives $a = -1$ and $b = -2$. Then $p(x) = q(x) \times (x^2 - x - 2) = q(x)(x - 2)(x + 1)$. The extra factor $(x - 2)(x + 1)$ contributes the zeroes 2 and -1, which belong to p(x) but not to q(x).

Answer: **$a = -1$, $b = -2$. The zeroes 1 and 2 of p(x) (specifically 2 and -1 from the extra quadratic factor) are not zeroes of q(x).**

Quick Revision Sheet (Night Before the Exam)

- Zeroes are where the graph meets the x-axis. Quadratic → up to 2 zeroes; cubic → up to 3.
- Quadratic ax^2+bx+c: sum of zeroes = $-b/a$, product = c/a.
- Cubic ax^3+bx^2+cx+d: sum = $-b/a$; sum two-at-a-time = c/a; product = $-d/a$.
- Build a quadratic from zeroes: $x^2 - (\text{sum})x + (\text{product})$. Multiply by any constant for equivalent answers.
- Division algorithm: $p(x) = g(x)q(x) + r(x)$, with $r(x) = 0$ or degree $r(x)$ < degree $g(x)$.
- Degree rule: degree of dividend = degree of divisor + degree of quotient (when remainder smaller).
- Equal zeroes ⇔ discriminant $b^2 - 4ac = 0$.
- Sign tricks: positive product = same-sign zeroes; negative product = opposite-sign zeroes; the sign of the sum then pins them down.
- To find unknown zeroes when one factor is given: divide it out, then factor or use the quadratic formula on what remains.