

CLASS 10 MATHEMATICS | NCERT EXEMPLAR | CHAPTER 6

TRIANGLES

Similarity, BPT and Pythagoras Theorem — Complete Study Guide with Diagrams

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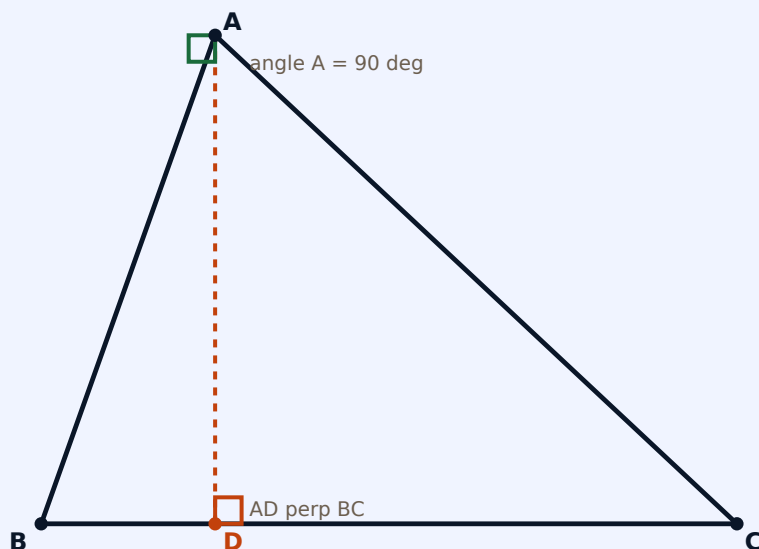
KEY CONCEPTS

AA Similarity	If 2 angles of one triangle = 2 angles of another -> triangles are similar.
SSS Similarity	All 3 pairs of corresponding sides are proportional.
SAS Similarity	2 pairs of sides proportional AND the included (between them) angle equal.
BPT	If $DE \parallel BC$ in triangle ABC, then $AD/DB = AE/EC$.
Area Ratio	Area ratio = (side ratio) ² . Areas scale as SQUARE of the side ratio.
Altitude in right tri	$BD \times CD = AD^2$ (where AD perpendicular BC and angle A = 90 deg).
Pythagoras	In right triangle: $a^2 + b^2 = c^2$ (hypotenuse ² = sum of squares of legs).
Perimeter Ratio	Perimeter ratio = side ratio (not squared — only areas use the square).

MCQ EXERCISE 6.1 — Multiple Choice Questions

- Q1** In the figure, angle $BAC = 90^\circ$ and AD perpendicular BC . Which relation is correct?

DIAGRAM — Q1



(A) $BD \times CD = BC^2$

(B) $AB \times AC = BC^2$

(C) [OK] $BD \times CD = AD^2$

(D) $AB \times AC = AD^2$

Solution — Q1

AD is the altitude from the right angle A to hypotenuse BC .

In a right triangle with altitude to hypotenuse: $\text{altitude}^2 = \text{product of two segments}$.

[OK] $BD \times CD = AD^2$

- Q2** Diagonals of a rhombus are 16 cm and 12 cm. Find the side.

(A) 9 cm

(B) [OK] 10 cm

(C) 8 cm

(D) 20 cm

Solution — Q2

Diagonals of a rhombus bisect each other at right angles.

Half-diagonals = 8 cm and 6 cm. $\text{Side}^2 = 8^2 + 6^2 = 100$.

[OK] Side = 10 cm

Q3 If triangle ABC ~ triangle EDF and NOT similar to triangle DEF, which is NOT true?

(A) $BC \times EF = AC \times FD$

(B) $AB \times EF = AC \times DE$

(C) [OK] $BC \times DE = AB \times EF$

(D) $BC \times DE = AB \times FD$

Solution — Q3

Correspondence: $A \leftrightarrow E$, $B \leftrightarrow D$, $C \leftrightarrow F$. Correct ratios: $AB/ED = BC/DF = CA/FE$.

Option (C) uses wrong pairing of sides.

[OK] Option (C) is NOT true

Q4 If $AB/QR = BC/PR = CA/PQ$, which similarity is correct?

(A) [OK] tri PQR ~ tri CAB

(B) tri PQR ~ tri ABC

(C) tri CBA ~ tri PQR

(D) tri BCA ~ tri PQR

Solution — Q4

Matching: $A \leftrightarrow Q$, $B \leftrightarrow R$, $C \leftrightarrow P$.

[OK] tri PQR ~ tri CAB

Q5 Segments AC and BD intersect at P. $PA=6, PB=3, PC=2.5, PD=5$, angle APB = 50 deg, angle CDP = 30 deg. Find angle PBA.

(A) 50 deg

(B) 30 deg

(C) 60 deg

(D) [OK] 100 deg

Solution — Q5

$PA/PD = 6/5$ and $PB/PC = 3/2.5 = 6/5$. Also angle APB = angle DPC (vertical).

tri PAB ~ tri PDC (SAS). So angle PBA = angle PCD.

In tri PDC: $180 - 50 - 30 = 100$ deg.

[OK] angle PBA = 100 deg

Q6 In tri DEF and tri PQR, angle D = angle Q and angle R = angle E. Which is NOT true?

(A) $EF/PR = DF/PQ$

(B) [OK] $DE/PQ = EF/PR$

(C) $DE/QR = DF/PQ$

(D) $EF/PR = DE/QR$

Solution — Q6

Correspondence: $D \leftrightarrow Q$, $E \leftrightarrow R$, $F \leftrightarrow P$. Correct: $DE/QR = EF/PR = FD/PQ$.

Option (B) says DE/PQ which is wrong — DE corresponds to QR.

[OK] Option (B) is NOT true

Q7 angle B = angle E, angle F = angle C, AB = 3DE. The two triangles are:

(A) Congruent but not similar

(B) [OK] Similar but not congruent

(C) Neither

(D) Both congruent and similar

Solution — Q7

By AA: $\triangle ABC \sim \triangle DEF$. But $AB = 3DE$ means sides in ratio 3:1 $\rightarrow$ not congruent.

[OK] Similar but not congruent

Q8 $\triangle ABC \sim \triangle PQR$ with $BC/QR = 1/3$. Find $\text{ar}(\triangle PQR)/\text{ar}(\triangle BCA)$.

(A) [OK] 9

(B) 3

(C) $1/3$

(D) $1/9$

Solution — Q8

Area ratio = (side ratio)² = $(1/3)^2 = 1/9$.

So $\text{ar}(\triangle PQR)/\text{ar}(\triangle BCA) = 9$.

[OK] 9

Q9 $\triangle ABC \sim \triangle DFE$, angle A = 30 deg, angle C = 50 deg, AB = 5, AC = 8, DF = 7.5. Which is true?

(A) DE = 12, angle F = 50 deg

(B) [OK] DE = 12, angle F = 100 deg

(C) EF = 12, angle D = 100 deg

(D) EF = 12, angle D = 30 deg

Solution — Q9

$A \leftrightarrow D$, $B \leftrightarrow F$, $C \leftrightarrow E$. angle F = $180 - 30 - 50 = 100$ deg.

Scale = $DF/AB = 1.5$. $DE = AC \times 1.5 = 12$ cm.

[OK] DE = 12 cm, angle F = 100 deg

Q10 In $\triangle ABC$ and $\triangle DEF$, $AB/DE = BC/FD$. When are they similar?

(A) angle B = angle E

(B) angle A = angle D

(C) [OK] angle B = angle D

(D) angle A = angle F

Solution — Q10

AB and BC are sides meeting at B; DE and FD meet at D.

For SAS similarity, the included angle must match.

[OK] angle B = angle D

Q11 $\triangle ABC \sim \triangle QRP$, $\frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle QRP)} = \frac{9}{4}$, $AB = 18$, $BC = 15$. Find PR .

(A) [OK] 10 cm

(B) 12 cm

(C) $\frac{20}{3}$ cm

(D) 8 cm

Solution — Q11

$A \leftrightarrow Q$, $B \leftrightarrow R$, $C \leftrightarrow P$. So BC corresponds to RP .

$\left(\frac{BC}{RP}\right)^2 = \frac{9}{4} \rightarrow \frac{BC}{RP} = \frac{3}{2} \rightarrow \frac{15}{RP} = \frac{3}{2} \rightarrow RP = 10$.

[OK] $PR = 10$ cm

Q12 S is on PQ of $\triangle PQR$ with $PS = QS = RS$. Which holds?

(A) $PR \times QR = RS^2$

(B) $QS^2 + RS^2 = QR^2$

(C) [OK] $PR^2 + QR^2 = PQ^2$

(D) $PS^2 + RS^2 = PR^2$

Solution — Q12

$PS = QS$ means S is midpoint of PQ . $PS = RS = QS = \frac{PQ}{2}$.

Midpoint of hypotenuse is equidistant from all vertices $\rightarrow$ angle $PRQ = 90^\circ$.

By Pythagoras: $PR^2 + QR^2 = PQ^2$.

[OK] $PR^2 + QR^2 = PQ^2$

T/F EXERCISE 6.2 — Short Answer with Reasoning

Q1 Is a triangle with sides 25 cm, 5 cm, 24 cm a right triangle?

ANSWER: FALSE

$25^2 = 625$, $5^2 + 24^2 = 25 + 576 = 601$ which is NOT equal to 625. Not a right triangle.

Q2 $\triangle DEF \sim \triangle RPQ$. Is angle $D =$ angle R and angle $F =$ angle P ?

ANSWER: FALSE

Correspondence: $D \leftrightarrow R$, $E \leftrightarrow P$, $F \leftrightarrow Q$. So angle $D =$ angle R (true) but angle $F =$ angle Q (NOT angle P).

Q3 $PQ = 12.5$, $PA = 5$, $BR = 6$, $PB = 4$. Is $AB \parallel QR$?

ANSWER: TRUE

$\frac{PA}{QA} = \frac{5}{7.5} = \frac{2}{3}$ and $\frac{PB}{BR} = \frac{4}{6} = \frac{2}{3}$. Equal ratios $\rightarrow AB \parallel QR$ by Converse of BPT.

Q4 BD and CE intersect at P ($BP=5, CP=6, PD=10, PE=12$). Is $\triangle PBC \sim \triangle PDE$?

ANSWER: TRUE

$PB/PD=1/2$, $PC/PE=1/2$, $\angle BPC=\angle DPE$ (vertical opp.) $\rightarrow$ SAS similarity.

Q5 $\triangle PQR$: $\angle P=55^\circ, \angle Q=25^\circ$. $\triangle MST$: $\angle M=100^\circ, \angle S=25^\circ$. Is $\triangle PQR \sim \triangle TSM$?

ANSWER: FALSE

In $\triangle PQR$: $\angle R=100^\circ$. In $\triangle MST$: $\angle T=55^\circ$. Correct: $\triangle PQR \sim \triangle STM$, not TSM.

Q6 Two quadrilaterals are similar if their corresponding angles are equal.

ANSWER: FALSE

Both conditions needed: equal angles AND proportional sides (e.g., square vs rectangle).

Q7 Two sides and perimeter of $\triangle 1 = 3\times$ those of $\triangle 2$. Are they similar?

ANSWER: TRUE

If two sides and perimeter are in ratio 3:1, the third side is also 3:1. SSS similarity.

Q8 Two right triangles with one equal acute angle — are they similar?

ANSWER: TRUE

Both have 90° + one equal acute angle $\rightarrow$ AA similarity. Yes, similar.

Q9 Ratio of altitudes of two similar triangles = $3/5$. Is area ratio = $6/5$?

ANSWER: FALSE

Area ratio = $(\text{altitude ratio})^2 = (3/5)^2 = 9/25$. NOT $6/5$.

Q10 D on QR of $\triangle PQR$, $PD \perp QR$. Is $\triangle PQD \sim \triangle RPD$?

ANSWER: FALSE

This requires $\angle P = 90^\circ$ in the original triangle, which is not given.

Q11 In figure, $\angle D = \angle C$. Is $\triangle ADE \sim \triangle ACB$?

ANSWER: TRUE

$\angle A$ is common to both. $\angle D = \angle C$ (given). By AA: $\triangle ADE \sim \triangle ACB$.

Q12 One angle equal AND two sides proportional $\rightarrow$ similar?

ANSWER: FALSE

Only if the equal angle is the INCLUDED angle between the proportional sides (SAS rule).

SA EXERCISE 6.3 — Short Answer Questions

Q1 In $\triangle PQR$, $PR^2 - PQ^2 = QR^2$ and $QM \perp PR$. Prove $QM^2 = PM \times MR$.

Proof — Q1

$PR^2 - PQ^2 = QR^2 \Rightarrow QR^2 + PQ^2 = PR^2 \Rightarrow \angle PQR = 90^\circ$ (converse Pythagoras).

In a right triangle, altitude to hypotenuse satisfies: altitude² = product of two segments.

$\Rightarrow QM^2 = PM \times MR$ (Proved)

Q2 Find x for $DE \parallel AB$. ($DC=x+3$, $AD=3x+19$, $EC=x$, $BE=3x+4$)

Solution — Q2

By BPT: $DC/AD = EC/BE \Rightarrow (x+3)/(3x+19) = x/(3x+4)$.

$(x+3)(3x+4) = x(3x+19) \Rightarrow 3x^2+13x+12 = 3x^2+19x \Rightarrow 6x=12$.

[OK] $x = 2$

Q4 Diagonals of trapezium PQRS intersect at O, $PQ \parallel RS$, $PQ=3RS$. Find $\text{ar}(\triangle POQ) : \text{ar}(\triangle ROS)$.

Solution — Q4

$PQ \parallel RS \Rightarrow \triangle POQ \sim \triangle ROS$ (AA). Scale = $PQ/RS = 3$.

[OK] $\text{ar}(\triangle POQ) : \text{ar}(\triangle ROS) = 9 : 1$

Q6 Find the altitude of an equilateral triangle of side 8 cm.

Solution — Q6

Altitude = $(\sqrt{3}/2) \times \text{side} = (\sqrt{3}/2) \times 8 = 4\sqrt{3}$ cm.

[OK] Altitude = $4\sqrt{3}$ cm (approx 6.93 cm)

Q7 $\triangle ABC \sim \triangle DEF$, $AB=4$, $DE=6$, $EF=9$, $FD=12$. Find perimeter of $\triangle ABC$.

Solution — Q7

Scale ratio: $AB/DE = 4/6 = 2/3$.

$BC = EF \times (2/3) = 6$ cm. $CA = FD \times (2/3) = 8$ cm.

[OK] Perimeter of $\triangle ABC = 4+6+8 = 18$ cm

Q8 $DE \parallel BC$ in figure ($DE=6\text{cm}$, $BC=12\text{cm}$). Find $\text{ar}(\text{ADE}) : \text{ar}(\text{DECB})$.

Solution — Q8

$DE \parallel BC \Rightarrow \text{tri ADE} \sim \text{tri ABC}$. Scale = $6/12 = 1/2$.

$\text{ar}(\text{ADE})/\text{ar}(\text{ABC}) = (1/2)^2 = 1/4$.

$\text{ar}(\text{DECB}) = \text{ar}(\text{ABC}) - \text{ar}(\text{ADE}) = 4 - 1 = 3$ parts.

[OK] $\text{ar}(\text{ADE}) : \text{ar}(\text{DECB}) = 1 : 3$

Q9 Trapezium ABCD, $AB \parallel DC$, $PQ \parallel DC$. $PD=18$, $BQ=35$, $QC=15$. Find AD.

Solution — Q9

By BPT: $AP/PD = BQ/QC \Rightarrow AP/18 = 35/15 = 7/3 \Rightarrow AP = 42$ cm.

[OK] $AD = AP + PD = 42 + 18 = 60$ cm

Q10 Sides of two similar triangles in ratio 2:3. Smaller area = 48 cm^2 . Find larger area.

Solution — Q10

Area ratio = $(2/3)^2 = 4/9$.

$48 / \text{Area}_{\text{large}} = 4/9 \Rightarrow \text{Area}_{\text{large}} = 108 \text{ cm}^2$.

[OK] Area of larger triangle = 108 cm^2

Q12 Areas of two similar triangles are 36 and 100 cm^2 . Larger side = 20 cm. Find smaller side.

Solution — Q12

Side ratio = $\sqrt{36/100} = 3/5$.

Smaller side = $20 \times (3/5) = 12$ cm.

[OK] Corresponding side = 12 cm

Q14 Tower 15m casts 24m shadow. Pole casts 16m shadow. Find pole height.

Solution — Q14

Similar triangles (same sun angle): $H_1/S_1 = H_2/S_2$.

$15/24 = h/16 \Rightarrow h = 10$ m.

[OK] Height of pole = 10 m

Q15 10m ladder, foot 6m from wall. Find height reached.

Solution — Q15

By Pythagoras: $h^2 + 6^2 = 10^2 \Rightarrow h^2 = 64 \Rightarrow h = 8$ m.

[OK] Height = 8 m

LA EXERCISE 6.4 — Long Answer Questions

Q1 angle $A = \text{angle } C$, $AB = 6$, $BP = 15$, $AP = 12$, $CP = 4$. Find PD and CD .

Solution — Q1

tri $ABP \sim \text{tri } CDP$ (AA: angle $A = \text{angle } C$, vertical angles equal).

$$AP/CP = BP/DP \Rightarrow 12/4 = 15/DP \Rightarrow DP = 5 \text{ cm.}$$

$$AP/CP = AB/CD \Rightarrow 12/4 = 6/CD \Rightarrow CD = 2 \text{ cm.}$$

[OK] $PD = 5 \text{ cm}$, $CD = 2 \text{ cm}$

Q2 tri $ABC \sim \text{tri } EDF$, $AB = 5$, $AC = 7$, $DF = 15$, $DE = 12$. Find remaining sides.

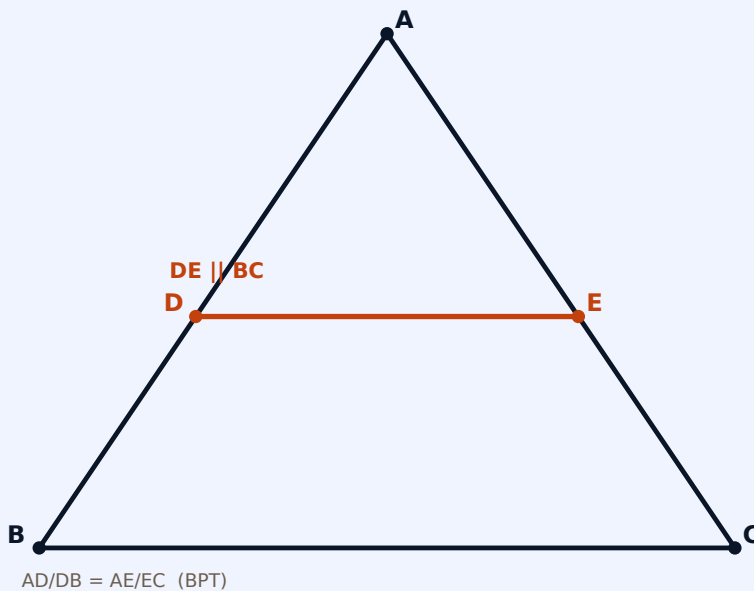
Solution — Q2

$A \leftrightarrow E$, $B \leftrightarrow D$, $C \leftrightarrow F$. Scale = $AB/ED = 5/12$.

$$BC = DF \times (5/12) = 6.25 \text{ cm. } EF = AC \times (12/5) = 16.8 \text{ cm.}$$

[OK] $BC = 6.25 \text{ cm}$, $EF = 16.8 \text{ cm}$

Q3 Prove BPT: A line parallel to one side of a triangle divides the other two sides proportionally.

DIAGRAM — Q3**Proof of BPT — Q3**

Given: $DE \parallel BC$ in tri ABC (D on AB , E on AC). Draw $DM \perp AC$, $EN \perp AB$. Join BE and CD .

$$\text{ar}(\text{tri } ADE)/\text{ar}(\text{tri } BDE) = AD/DB \dots(1)$$

$$\text{ar}(\text{tri } ADE)/\text{ar}(\text{tri } CDE) = AE/EC \dots(2)$$

Since $DE \parallel BC$: $\text{ar}(\text{tri } BDE) = \text{ar}(\text{tri } CDE)$ (same base, same height) $\dots(3)$

$\Rightarrow$ From (1),(2),(3): $AD/DB = AE/EC$ (Proved)

Q5 5m ladder reaches 4m high. Foot moved 1.6m closer. How far does top slide up?

Solution — Q5

Initial base = $\sqrt{5^2 - 4^2} = 3\text{m}$. New base = $3 - 1.6 = 1.4\text{m}$.

New height = $\sqrt{25 - 1.96} = \sqrt{23.04} = 4.8\text{m}$.

[OK] Top slides up by $4.8 - 4 = 0.8\text{ m}$

Q6 AC perp CB, $AC=2x\text{ km}$, $CB=2(x+7)\text{ km}$, highway $AB=26\text{ km}$. Find distance saved.

Solution — Q6

$(2x)^2 + (2(x+7))^2 = 26^2 \Rightarrow x^2 + (x+7)^2 = 169$.

$2x^2 + 14x + 49 = 169 \Rightarrow x^2 + 7x - 60 = 0 \Rightarrow (x+12)(x-5)=0 \Rightarrow x=5$.

$AC=10$, $CB=24$. Old route= 34km . Highway= 26km .

[OK] Distance saved = $34 - 26 = 8\text{ km}$

Q7 Flag pole 18m, shadow 9.6m. Find distance from top to far end of shadow.

Solution — Q7

Distance = $\sqrt{18^2 + 9.6^2} = \sqrt{324 + 92.16} = \sqrt{416.16}$ approx 20.4 m .

[OK] Distance approx 20.4 m

Q9 tri ABC right-angled at B, BD perp AC, $AD=4\text{cm}$, $CD=5\text{cm}$. Find BD and AB.

Solution — Q9

$BD^2 = AD \times CD = 4 \times 5 = 20 \Rightarrow BD = 2\sqrt{5}\text{ cm}$.

$AB^2 = AD \times AC = 4 \times 9 = 36 \Rightarrow AB = 6\text{ cm}$.

[OK] $BD = 2\sqrt{5}\text{ cm}$, $AB = 6\text{ cm}$

Q10 tri PQR right-angled at Q, QS perp PR, $PQ=6$, $PS=4$. Find QS, RS, QR.

Solution — Q10

$PQ^2 = PS \times PR \Rightarrow 36 = 4 \times PR \Rightarrow PR = 9\text{cm}$. $RS = 9-4 = 5\text{cm}$.

$QS^2 = PS \times RS = 20 \Rightarrow QS = 2\sqrt{5}\text{ cm}$.

$QR^2 = RS \times PR = 45 \Rightarrow QR = 3\sqrt{5}\text{ cm}$.

[OK] $QS = 2\sqrt{5}$, $RS = 5\text{ cm}$, $QR = 3\sqrt{5}\text{ cm}$

REVISION QUICK REVISION — ANSWER KEY

Ex 6.1 MCQ	Q1->C, Q2->B, Q3->C, Q4->A, Q5->D, Q6->B, Q7->B, Q8->A
Ex 6.1 MCQ	Q9->B, Q10->C, Q11->A, Q12->C
Ex 6.2 T/F	Q1:No, Q2:No, Q3:Yes, Q4:Yes, Q5:No, Q6:No, Q7:Yes, Q8:Yes

Ex 6.2 T/F	Q9:No, Q10:No, Q11:Yes, Q12:No
Ex 6.3	Q2: $x=2$, Q4:9:1, Q6: $4\sqrt{3}$ cm, Q7:18cm, Q8:1:3, Q9:60cm
Ex 6.3	Q10: 108cm^2 , Q12:12cm, Q14:10m, Q15:8m
Ex 6.4	Q1:PD=5,CD=2cm Q2:BC=6.25,EF=16.8cm Q5:0.8m Q6:8km
Ex 6.4	Q7:20.4m Q9:BD= $2\sqrt{5}$,AB=6cm Q10:QS= $2\sqrt{5}$,RS=5,QR= $3\sqrt{5}$

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