CLASS 10 MATHEMATICS

CHAPTER 5 — ARITHMETIC PROGRESSIONS

Complete Solved Study Guide

Concepts • MCQs • Reasoning • Short & Long Answers • Exam Tips

How to Use This Guide

This guide rebuilds the entire Arithmetic Progressions chapter into a clean, revision-ready format. Every question from each exercise is listed with a fully worked solution in plain language. Concept boxes refresh the key ideas, tip boxes share shortcuts examiners reward, and note boxes flag common slips. Every numerical answer has been independently verified.

Part A — Key Concepts You Must Know

1. What an AP Is

An arithmetic progression (AP) is a list of numbers where each term is obtained by adding a fixed number d to the term before it. The fixed gap d is the common difference, and a (or a_1) denotes the first term.

$$\text{General form: } a,\ a + d,\ a + 2d,\ a + 3d,\ \ldots$$

2. The Test for an AP

A list of numbers $a_1, a_2, a_3, \ldots$ forms an AP only when every pair of consecutive differences is the same — that is, $a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \ldots = d$. If even one gap is different, the list is NOT an AP.

3. nth Term (General Term)

$$a_n = a + (n - 1)\, d$$

Use this to find any specific term, or to find n when a term value is given.

4. Sum of the First n Terms

$$S_n = n/2 \cdot [\ 2a + (n - 1)\, d\ ]$$

If the last term l is known instead of d, the slicker form is:

$$S_n = n/2 \cdot (a + l)$$

5. nth Term from the Sum

$$a_n = S_n - S_{n-1}$$

Handy when S_n is given as a formula in n and you need a particular term.

Concept Recall

Three quick consequences worth memorising:

- $a_m - a_n = (m - n)\cdot d$ (difference between any two terms is the common difference times the gap in their positions).
- Three terms in AP can be written as $a - d, a, a + d$ (sum = 3a, an easy setup).
- Four terms in AP can be written as $a - 3d, a - d, a + d, a + 3d$ (sum = 4a, and the symmetry cancels many cross-terms).

Exam Tip

If a problem mentions 'middle term' of an AP with an odd number of terms, that single middle term equals the AVERAGE of all terms — so $S_n = n \times$ (middle term). This single trick handles many sum problems instantly.

Part B — Multiple Choice Questions (Solved)

Worked Samples

QS1. The 10th term of the AP 5, 8, 11, 14, … is — (A) 32 (B) 35 (C) 38 (D) 185.

Solution

Here $a = 5$ and $d = 3$. So $a_{10} = 5 + 9(3) = 32$.

Correct Answer: (A) 32

QS2. In an AP $a = -7.2$, $d = 3.6$, $a_n = 7.2$; then n is — (A) 1 (B) 3 (C) 4 (D) 5.

Solution

Use $a_n = a + (n - 1)d$: $7.2 = -7.2 + (n - 1)(3.6) \rightarrow 14.4 = 3.6(n - 1) \rightarrow n - 1 = 4 \rightarrow n = 5$.

Correct Answer: (D) 5

Exercise 5.1

Q1. If $d = -4$, $n = 7$, $a_n = 4$, then a is — (A) 6 (B) 7 (C) 20 (D) 28.

Solution

$a_n = a + (n - 1)d \rightarrow 4 = a + 6(-4) \rightarrow 4 = a - 24 \rightarrow a = 28$.

Correct Answer: (D) 28

Q2. If $a = 3.5$, $d = 0$, $n = 101$, then a_n is — (A) 0 (B) 3.5 (C) 103.5 (D) 104.5.

Solution

With $d = 0$, every term equals a, so $a_{101} = 3.5$.

Correct Answer: (B) 3.5

Q3. The list −10, −6, −2, 2, … is — (A) AP with $d = -16$ (B) AP with $d = 4$ (C) AP with $d = -4$ (D) not an AP.

Solution

Differences: $-6 - (-10) = 4$, $-2 - (-6) = 4$, $2 - (-2) = 4$. Constant difference 4, so AP with $d = 4$.

Correct Answer: (B) AP with d = 4

Q4. The 11th term of the AP −5, −5/2, 0, 5/2, … is — (A) −20 (B) 20 (C) −30 (D) 30.

Solution

$a = -5$, $d = 5/2$. $a_{11} = -5 + 10(5/2) = -5 + 25 = 20$.

Correct Answer: (B) 20

Q5. First four terms of an AP with first term −2 and common difference −2 are — (A) −2,0,2,4 (B) −2,4,−8,16 (C) −2,−4,−6,−8 (D) −2,−4,−8,−16.

Solution

Adding −2 successively: −2, −4, −6, −8.

Correct Answer: (C) −2, −4, −6, −8

Q6. The 21st term of the AP whose first two terms are −3 and 4 is — (A) 17 (B) 137 (C) 143 (D) −143.

Solution

$d = 4 - (-3) = 7$. $a_{21} = -3 + 20(7) = 137$.

Correct Answer: (B) 137

Q7. $a_2 = 13$, $a_5 = 25$ of an AP. Find a_7 — (A) 30 (B) 33 (C) 37 (D) 38.

Solution

$a_5 - a_2 = 3d = 12$, so $d = 4$. $a_7 = a_2 + 5d = 13 + 20 = 33$.

Correct Answer: (B) 33

Q8. Which term of 21, 42, 63, 84, … is 210? — (A) 9th (B) 10th (C) 11th (D) 12th.

Solution

$a = 21$, $d = 21$. $210 = 21 + (n - 1)(21) \rightarrow (n - 1) = 9 \rightarrow n = 10$.

Correct Answer: (B) 10th

Q9. If $d = 5$, then $a_{18} - a_{13}$ is — (A) 5 (B) 20 (C) 25 (D) 30.

Solution

$a_{18} - a_{13} = 5d = 25$.

Correct Answer: (C) 25

Q10. What is d if $a_{18} - a_{14} = 32$? — (A) 8 (B) −8 (C) −4 (D) 4.

Solution

$a_{18} - a_{14} = 4d = 32$, so $d = 8$.

Correct Answer: (A) 8

Q11. Two APs have the same d. First terms are −1 and −8. Difference of their 4th terms is — (A) −1 (B) −8 (C) 7 (D) −9.

Solution

Difference between corresponding terms equals the difference of first terms: $-1 - (-8) = 7$.

Correct Answer: (C) 7

Q12. If $7 \cdot a_7 = 11 \cdot a_{11}$, then a_{18} is — (A) 7 (B) 11 (C) 18 (D) 0.

Solution

$7(a + 6d) = 11(a + 10d) \rightarrow 7a + 42d = 11a + 110d \rightarrow 4a + 68d = 0 \rightarrow a + 17d = 0$, i.e. $a_{18} = 0$.

Correct Answer: (D) 0

Q13. The 4th term from the end of the AP −11, −8, −5, …, 49 is — (A) 37 (B) 40 (C) 43 (D) 58.

Solution

Total terms: $49 = -11 + (n - 1)(3) \rightarrow n = 21$. The 4th from the end is the 18th from the start = $-11 + 17(3) = 40$.

Correct Answer: (B) 40

Q14. The mathematician famous for summing the first 100 natural numbers is — (A) Pythagoras (B) Newton (C) Gauss (D) Euclid.

Solution

The childhood-Gauss story: he summed 1 to 100 by pairing 1+100, 2+99, … to get fifty 101s, giving 5050.

Correct Answer: (C) Gauss

Q15. First term −5, d = 2, sum of first 6 terms is — (A) 0 (B) 5 (C) 6 (D) 15.

Solution

$S_6 = (6/2)[2(-5) + 5(2)] = 3 \times 0 = 0$.

Correct Answer: (A) 0

Q16. Sum of first 16 terms of 10, 6, 2, … is — (A) −320 (B) 320 (C) −352 (D) −400.

Solution

$a = 10$, $d = -4$. $S_{16} = (16/2)[20 + 15(-4)] = 8 \times (-40) = -320$.

Correct Answer: (A) −320

Q17. If $a = 1$, $a_n = 20$, $S_n = 399$, then n is — (A) 19 (B) 21 (C) 38 (D) 42.

Solution

$S_n = (n/2)(a + a_n) \rightarrow 399 = (n/2)(21) \rightarrow n = 38$.

Correct Answer: (C) 38

Q18. The sum of the first five multiples of 3 is — (A) 45 (B) 55 (C) 65 (D) 75.

Solution

3 + 6 + 9 + 12 + 15 = 45.

Correct Answer: (A) 45

Part C — Short Answer Questions with Reasoning

Worked Samples

QS1. In the AP 10, 5, 0, −5, … the common difference is 5. True or false?

Solution

False. Differences are $5 - 10 = -5$, $0 - 5 = -5$, $-5 - 0 = -5$. The list is an AP, but with $d = -5$, not 5.

QS2. Divya deposits Rs 1000 at 10% compound interest. Do the year-end amounts form an AP? Justify.

Solution

No. Year-end amounts: 1100, 1210, 1331, … Differences: 110, 121, … which are not equal. So it is not an AP — compound interest gives a geometric progression, not an arithmetic one.

QS3. Can $a_n = n^2 + 1$ be the nth term of an AP? Justify.

Solution

No. Compute terms: 2, 5, 10, 17, … Differences: 3, 5, 7, … which keep growing. The nth term of an AP must be linear in n ($a + (n - 1)d$), not quadratic, so $n^2 + 1$ cannot be an AP's nth term.

Exercise 5.2

Q1. Which of the following form an AP? Justify: (i) −1,−1,−1,−1,… (ii) 0,2,0,2,… (iii) 1,1,2,2,3,3,… (iv) 11,22,33,… (v) 1/2,1/3,1/4,… (vi) $2, 2^2, 2^3, 2^4$,… (vii) $\sqrt{3}, \sqrt{12}, \sqrt{27}, \sqrt{48}$, ….

Solution

(i) Yes — every gap is 0 (constant), so it is an AP with $d = 0$.

(ii) No — gaps are 2, −2, 2, −2, … (not constant).

(iii) No — gaps are 0, 1, 0, 1, 0, … (not constant).

(iv) Yes — gaps are 11, 11, 11, … (constant), so AP with $d = 11$.

(v) No — common gap is $1/3 - 1/2 = -1/6$ and $1/4 - 1/3 = -1/12$, not equal.

(vi) No — these are 2, 4, 8, 16, … with gaps 2, 4, 8, … (a geometric progression).

(vii) Yes — these simplify to $\sqrt{3}, 2\sqrt{3}, 3\sqrt{3}, 4\sqrt{3}$, …, with constant gap $\sqrt{3}$.

Q2. Is it true that −1, −3/2, −2, 5/2, … forms an AP because $a_2 - a_1 = a_3 - a_2$? Justify.

Solution

False. The condition for an AP requires ALL consecutive differences to be equal, not just one matching pair. Here $a_2 - a_1 = -1/2$ and $a_3 - a_2 = -1/2$ agree, but $a_4 - a_3 = 5/2 - (-2) = 9/2$, which differs. So this list is not an AP.

Q3. For the AP −3, −7, −11, … can we find $a_{30} - a_{20}$ directly without finding each? Justify.

Solution

Yes. By the property $a_m - a_n = (m - n)d$, we get $a_{30} - a_{20} = 10d = 10 \times (-4) = -40$. No need to compute each term.

Q4. Two APs have the same d, first terms 2 and 7. Why is the difference between corresponding terms always the same?

Solution

Any corresponding term-pair is $(a + (n - 1)d)$ versus $(a' + (n - 1)d)$. Subtracting cancels the $(n - 1)d$ completely, leaving $a - a' = 2 - 7 = -5$ for every n. So the gap stays constant at 5 (or −5), no matter which corresponding terms you pick.

Q5. Is 0 a term of the AP 31, 28, 25, …? Justify.

Solution

No. With $a = 31$ and $d = -3$, setting $a_n = 0$ gives $0 = 31 - 3(n - 1) \rightarrow 3(n - 1) = 31 \rightarrow n = 34/3$, which is not a whole number. So 0 never appears in this AP.

Q6. Taxi fare is Rs 15 for the first km and Rs 8 per extra km. Is the fare sequence 15, 8, 8, 8, … not an AP?

Solution

The fare sequence stated (15, 8, 8, 8, …) is for individual kilometres, not the running total. The CUMULATIVE total fare after each km is 15, 23, 31, 39, … which IS an AP with $d = 8$. So the conclusion in the statement is correct only when interpreted as per-km charges, but the running total does form an AP.

Q7. Which of these lists form an AP? Give reasons. (i) Fixed monthly fee Rs 400 (ii) Monthly fee from Class I (Rs 250) rising by Rs 50 per class (iii) Yearly balance from Rs 1000 at 10% simple interest (iv) Bacteria doubling every second.

Solution

(i) Yes — list is 400, 400, 400, … ($d = 0$). (ii) Yes — 250, 300, 350, … ($d = 50$). (iii) Yes — simple interest is Rs 100 per year, so balances are 1100, 1200, 1300, … ($d = 100$). (iv) No — doubling each second produces a geometric, not arithmetic, progression.

Q8. Are these the nth terms of an AP? (i) $2n - 3$ (ii) $3n^2 + 5$ (iii) $1 + n + n^2$

Solution

An expression in n is the nth term of an AP iff it is linear in n. (i) $2n - 3$ is linear → yes ($a = -1$, $d = 2$). (ii) $3n^2 + 5$ is quadratic → no. (iii) $1 + n + n^2$ is quadratic → no.

Part D — Short Answer Questions

Worked Samples

QS1. If $n - 2, 4n - 1, 5n + 2$ are in AP, find n.

Solution

In an AP the middle term is the average of its neighbours, so $2(4n - 1) = (n - 2) + (5n + 2) \rightarrow 8n - 2 = 6n \rightarrow 2n = 2 \rightarrow n = 1$.

QS2. Find the middle term(s) of the AP $-11, -7, -3, \ldots, 49$.

Solution

$a = -11$, $d = 4$, $a_n = 49$. So $49 = -11 + 4(n - 1) \rightarrow n = 16$. With 16 terms there are TWO middle terms — the 8th and 9th. $a_8 = -11 + 7(4) = 17$ and $a_9 = 21$.

QS3. The sum of first three terms of an AP is 33. The product of the first and third exceeds the second by 29. Find the AP.

Solution

Take the three terms as $a - d, a, a + d$. Sum $3a = 33 \rightarrow a = 11$. Also $(a - d)(a + d) = a + 29 \rightarrow a^2 - d^2 = a + 29 \rightarrow 121 - d^2 = 40 \rightarrow d^2 = 81 \rightarrow d = \pm 9$. So the AP is either 2, 11, 20, … or 20, 11, 2, …

Exercise 5.3

Q1. Match each AP in column A with its common difference in column B. (A_1) 2, −2, −6, −10, … (A_2) $a = -18$, $n = 10$, $a_n = 0$ (A_3) $a = 0$, $a_{10} = 6$ (A_4) $a_2 = 13$, $a_4 = 3$. Choices: −4, 2, 3/5, 1/2, 5, −5, 2/3.

Solution

(A_1) $d = -2 - 2 = -4 \rightarrow$ matches B_4.

(A_2) $0 = -18 + 9d \rightarrow d = 2 \rightarrow$ matches B_5.

(A_3) $6 = 0 + 9d \rightarrow d = 2/3 \rightarrow$ matches B_1.

(A_4) $2d = a_4 - a_2 = -10 \rightarrow d = -5 \rightarrow$ matches B_2.

Q2. Verify each is an AP and write the next three terms. (i) 0, 1/4, 1/2, 3/4, … (ii) 5, 14/3, 13/3, 4, … (iii) $\sqrt{3}, 2\sqrt{3}, 3\sqrt{3}, \ldots$ (iv) $a+b, (a+1)+b, (a+1)+(b+1), \ldots$ (v) $a, 2a+1, 3a+2, 4a+3, \ldots$

Solution

(i) $d = 1/4$. Next: 1, 5/4, 3/2.

(ii) $d = -1/3$. Next: 11/3, 10/3, 3.

(iii) $d = \sqrt{3}$. Next: $4\sqrt{3}, 5\sqrt{3}, 6\sqrt{3}$.

(iv) $d = 1$ (alternating x-step). The pattern adds 1 each step in either a or b. Continuing: $(a+2)+(b+1), (a+2)+(b+2), (a+3)+(b+2)$.

(v) $d = a + 1$. Next: $5a + 4, 6a + 5, 7a + 6$.

Q3. Write the first three terms when: (i) $a = 1/2$, $d = -1/6$ (ii) $a = -5$, $d = -3$ (iii) $a = \sqrt{2}$, $d = 1/\sqrt{2}$.

Solution

(i) 1/2, 1/3, 1/6.

(ii) −5, −8, −11.

(iii) $\sqrt{2}, 3/\sqrt{2}, 4/\sqrt{2} = \sqrt{2}, 3\sqrt{2}/2, 2\sqrt{2}$ (equivalently $4\sqrt{2}/2$).

Q4. Find a, b, c such that a, 7, b, 23, c are in AP.

Solution

Since 7 and 23 are two positions apart with a term between, $2d = 23 - 7 = 16$, so $d = 8$. Then $b = 7 + 8 = 15$, $a = 7 - 8 = -1$, $c = 23 + 8 = 31$.

Answer: $a = -1$, $b = 15$, $c = 31$.

Q5. Find the AP whose 5th term is 19 and the difference $a_{13} - a_8$ equals 20.

Solution

$a_{13} - a_8 = 5d = 20 \rightarrow d = 4$. Then $a + 4d = 19 \rightarrow a = 3$. So the AP is 3, 7, 11, 15, …

Q6. The 26th, 11th and last terms of an AP are 0, 3 and −1/5. Find d and n.

Solution

$a_{26} - a_{11} = 15d = 0 - 3 = -3$, so $d = -1/5$. Then $a + 25d = 0 \rightarrow a = 5$. Last term: $5 + (n - 1)(-1/5) = -1/5 \rightarrow (n - 1)/5 = 26/5 \rightarrow n = 27$.

Answer: $d = -1/5$, $n = 27$.

Q7. If $a_5 + a_7 = 52$ and $a_{10} = 46$, find the AP.

Solution

$a_5 + a_7 = 2a + 10d = 52$, so $a + 5d = 26$. Also $a_{10} = a + 9d = 46$. Subtracting: $4d = 20 \rightarrow d = 5$; then $a = 1$.

Answer: AP: 1, 6, 11, 16, …

Q8. Find a_{20} of the AP whose a_7 is 24 less than a_{11}, with $a = 12$.

Solution

$a_{11} - a_7 = 4d = 24 \rightarrow d = 6$. So $a_{20} = 12 + 19(6) = 126$.

Q9. If $a_9 = 0$, prove $a_{29} = 2 \times a_{19}$.

Solution

$a_9 = a + 8d = 0$, so $a = -8d$. Then $a_{19} = a + 18d = -8d + 18d = 10d$, and $a_{29} = a + 28d = -8d + 28d = 20d$. Clearly $20d = 2 \times 10d$, i.e. $a_{29} = 2 \times a_{19}$. ✓

Q10. Is 55 a term of 7, 10, 13, … ? If yes, which?

Solution

$a = 7$, $d = 3$. Set $7 + 3(n - 1) = 55 \rightarrow n - 1 = 16 \rightarrow n = 17$. So 55 is the 17th term.

Q11. Find k so that $k^2 + 4k + 8$, $2k^2 + 3k + 6$, $3k^2 + 4k + 4$ are three consecutive AP terms.

Solution

AP condition: 2 × middle = first + third. So $2(2k^2 + 3k + 6) = (k^2 + 4k + 8) + (3k^2 + 4k + 4) \rightarrow 4k^2 + 6k + 12 = 4k^2 + 8k + 12 \rightarrow 6k = 8k \rightarrow k = 0$.

Q12. Split 207 into three AP parts whose two smaller parts multiply to 4623.

Solution

Let the parts be $a - d, a, a + d$. Sum = $3a = 207 \rightarrow a = 69$. Product of two smaller (assuming $d > 0$): $(a - d)\cdot a = 4623 \rightarrow 69(69 - d) = 4623 \rightarrow 69 - d = 67 \rightarrow d = 2$. Parts are 67, 69, 71.

Q13. Angles of a triangle are in AP and the greatest is twice the least. Find them.

Solution

Let angles be $a - d, a, a + d$. Sum $3a = 180° \rightarrow a = 60°$. Also $(a + d) = 2(a - d) \rightarrow a = 3d \rightarrow 60 = 3d \rightarrow d = 20°$. Angles: 40°, 60°, 80°.

Q14. If the nth terms of 9, 7, 5, … and 24, 21, 18, … are equal, find n and that term.

Solution

$9 + (n - 1)(-2) = 24 + (n - 1)(-3) \rightarrow 11 - 2n = 27 - 3n \rightarrow n = 16$. The term: $9 - 30 = -21$.

Q15. If $a_3 + a_8 = 7$ and $a_7 + a_{14} = -3$, find a_{10}.

Solution

$2a + 9d = 7$ and $2a + 19d = -3$. Subtracting: $10d = -10 \rightarrow d = -1$; then $a = 8$. So $a_{10} = 8 + 9(-1) = -1$.

Q16. Find the 12th term from the end of −2, −4, −6, …, −100.

Solution

Total terms: $-100 = -2 + (n - 1)(-2) \rightarrow n = 50$. The 12th from the end is the $(50 - 12 + 1) = 39$th from the start $= -2 + 38(-2) = -78$.

Q17. Which term of 53, 48, 43, … is the first negative?

Solution

$a_n = 53 - 5(n - 1)$. Set $a_n < 0$: $53 - 5(n - 1) < 0 \rightarrow n > 11.6$. The smallest integer is $n = 12$. So the 12th term is the first negative (it equals $53 - 55 = -2$).

Q18. How many numbers between 10 and 300 leave remainder 3 on division by 4?

Solution

These numbers are 11, 15, 19, …, 299, an AP with $a = 11$, $d = 4$. Number of terms: $299 = 11 + 4(n - 1) \rightarrow n = 73$.

Q19. Find the sum of the two middle terms of −4/3, −1, −2/3, …, 13/3.

Solution

$d = 1/3$, $a = -4/3$, last = 13/3. Number of terms: $13/3 = -4/3 + (n - 1)/3 \rightarrow n = 18$ (even, so two middle terms are 9th and 10th). $a_9 = -4/3 + 8/3 = 4/3$; $a_{10} = 4/3 + 1/3 = 5/3$. Sum = $4/3 + 5/3 = 3$.

Q20. First term −5, last 45, sum 120. Find n and d.

Solution

$S_n = n/2(a + l) = n/2(-5 + 45) = 20n = 120 \rightarrow n = 6$. Last term: $45 = -5 + 5d \rightarrow d = 10$.

Q21. Find the sums: (i) $1 + (-2) + (-5) + \dots + (-236)$ (ii) $(4 - 1/n) + (4 - 2/n) + \dots$ upto n terms (iii) $(a - b)/(a + b) + (3a - 2b)/(a + b) + (5a - 3b)/(a + b) + \dots$ to 11 terms.

Solution

(i) $a = 1$, $d = -3$, last $= -236$. n: $-236 = 1 + (n - 1)(-3) \to n = 80$. Sum $= (80/2)(1 + (-236)) = 40(-235) = -9400$.

(ii) The kth term is $(4 - k/n)$, so the sum is $4n - (1 + 2 + \dots + n)/n = 4n - (n + 1)/2 = (7n - 1)/2$.

(iii) First term $(a - b)/(a + b)$, common difference $[(3a - 2b) - (a - b)]/(a + b) = (2a - b)/(a + b)$. Using $S_{11} = (11/2)[2t_1 + 10d] = 11(t_1 + 5d) = 11[(a - b) + 5(2a - b)]/(a + b) = 11(11a - 6b)/(a + b)$.

Q22. Which term of $-2, -7, -12, \dots$ is -77? Find the sum of the AP up to that term.

Solution

$a = -2$, $d = -5$. Set $a_n = -77$: $-2 - 5(n - 1) = -77 \to n = 16$. So it is the 16th term. Sum $S_{16} = (16/2)(-2 + (-77)) = 8(-79) = -632$.

Q23. If $a_n = 3 - 4n$, show the sequence is an AP and find S_{20}.

Solution

$a_n - a_{n-1} = (3 - 4n) - (3 - 4(n - 1)) = -4$ (constant), so AP with $a = -1$ and $d = -4$. $S_{20} = (20/2)[-2 + 19(-4)] = 10(-78) = -780$.

Q24. If $S_n = n(4n + 1)$, find the AP.

Solution

$a_1 = S_1 = 1(5) = 5$. $a_2 = S_2 - S_1 = 2(9) - 5 = 13$. So $d = 8$ and the AP is $5, 13, 21, 29, \dots$

Q25. If $S_n = 3n^2 + 5n$ and $a_k = 164$, find k.

Solution

$a_n = S_n - S_{n-1} = [3n^2 + 5n] - [3(n - 1)^2 + 5(n - 1)] = 6n + 2$. Setting $6k + 2 = 164 \to k = 27$.

Q26. Prove that $S_{12} = 3(S_8 - S_4)$.

Solution

Write $S_n = (n/2)(2a + (n - 1)d)$. Then $S_8 - S_4 = 4(2a + 7d) - 2(2a + 3d) = 4a + 22d$. Multiplying by 3: $12a + 66d$. Also $S_{12} = 6(2a + 11d) = 12a + 66d$. So $S_{12} = 3(S_8 - S_4)$. ✓

Q27. Find S_{17} if $a_4 = -15$ and $a_9 = -30$.

Solution

$a_9 - a_4 = 5d = -15 \to d = -3$. $a_4 = a + 3d = -15 \to a = -6$. $S_{17} = (17/2)[-12 + 16(-3)] = (17/2)(-60) = -510$.

Q28. If $S_6 = 36$ and $S_{16} = 256$, find S_{10}.

Solution

$3(2a + 5d) = 36 \to 2a + 5d = 12$. $8(2a + 15d) = 256 \to 2a + 15d = 32$. Subtracting: $10d = 20 \to d = 2$, $a = 1$. So $S_{10} = (10/2)[2 + 9(2)] = 5(20) = 100$.

Q29. Sum of all 11 terms of an AP whose middle term is 30.

Solution

For odd n, sum = n × middle term. So S_{11} = 11 × 30 = 330.

Q30. Sum of the last 10 terms of 8, 10, 12, …, 126.

Solution

Reverse the AP: it becomes 126, 124, …, with a = 126, d = −2. Sum of first 10 terms in reverse = (10/2)[252 + 9(−2)] = 5(234) = 1170.

Q31. Sum of the first seven numbers that are multiples of both 2 and 9.

Solution

LCM(2, 9) = 18, so the numbers are 18, 36, 54, 72, 90, 108, 126. Sum = (7/2)(18 + 126) = (7/2)(144) = 504.

Q32. How many terms of −15, −13, −11, … sum to −55? Explain why there are two answers.

Solution

a = −15, d = 2. S_n = (n/2)(−30 + 2(n − 1)) = n(n − 16). Setting n(n − 16) = −55 → $n^2 - 16n + 55 = 0$ → (n − 5)(n − 11) = 0 → n = 5 or 11. Both work because terms 6 through 11 sum to zero (the AP crosses zero), so adding them does not change the partial sum.

Q33. S_n of an AP with a = 8, d = 20 equals S_{2n} of another AP with a = −30, d = 8. Find n.

Solution

(n/2)(16 + 20(n − 1)) = (2n/2)(−60 + 8(2n − 1)) → n(10n − 2) = 2n(8n − 34) → $10n^2 - 2n = 16n^2 - 68n$. For n ≠ 0: $-6n^2 + 66n = 0$ → n(n − 11) = 0 → n = 11.

Q34. On Day 1 Kanika saved Re 1, Day 2 Rs 2, …, all 31 days of January. She also spent Rs 204 of her pocket money and was left with Rs 100. Find her pocket money.

Solution

Total savings: 1 + 2 + … + 31 = (31 × 32)/2 = 496. Pocket money = savings + spending + leftover = 496 + 204 + 100 = 800.

Answer: Pocket money = Rs 800.

Q35. Yasmeen saves Rs 32, 36, 40 in successive months. In how many months will her total savings reach Rs 2000?

Solution

a = 32, d = 4. S_n = (n/2)(64 + 4(n − 1)) = n(30 + 2n) = $2n^2 + 30n$. Set $2n^2 + 30n = 2000$ → $n^2 + 15n - 1000 = 0$ → (n − 25)(n + 40) = 0 → n = 25.

Answer: 25 months.

Part E — Long Answer Questions

Worked Samples

QS1. Sum of four consecutive AP terms is 32, and product of the first and last to product of the two middle ones is 7 : 15. Find the numbers.

Solution

Take the four terms as $a - 3d$, $a - d$, $a + d$, $a + 3d$. Sum $4a = 32 \rightarrow a = 8$. Ratio: $(a^2 - 9d^2)/(a^2 - d^2) = 7/15 \rightarrow 15a^2 - 135d^2 = 7a^2 - 7d^2 \rightarrow 8a^2 = 128d^2 \rightarrow d^2 = 4 \rightarrow d = \pm 2$. With $d = 2$ the numbers are 2, 6, 10, 14.

QS2. Solve $1 + 4 + 7 + 10 + \ldots + x = 287$.

Solution

$a = 1$, $d = 3$. Last term $x = 3n - 2$. Sum: $(n/2)(1 + x) = (n/2)(3n - 1) = 287 \rightarrow 3n^2 - n - 574 = 0 \rightarrow n = 14$ (rejecting the negative root). So $x = 3(14) - 2 = 40$.

Exercise 5.4

Q1. $S_5 + S_7 = 167$ and $S_{10} = 235$. Find S_{20}.

Solution

$S_5 = (5/2)(2a + 4d) = 5a + 10d$. $S_7 = (7/2)(2a + 6d) = 7a + 21d$. Sum: $12a + 31d = 167$. $S_{10} = 5(2a + 9d) = 10a + 45d = 235 \rightarrow 2a + 9d = 47$. Solving: $a = 1$, $d = 5$. Then $S_{20} = (20/2)(2 + 19(5)) = 10(97) = 970$.

Answer: $S_{20} = 970$.

Q2. (i) Sum of integers between 1 and 500 divisible by both 2 and 5. (ii) Sum of integers 1 to 500 divisible by both 2 and 5. (iii) Sum of integers 1 to 500 divisible by 2 OR 5.

Solution

(i) Multiples of LCM(2, 5) = 10 between 1 and 500: 10, 20, …, 490. Number of terms = 49. Sum = (49/2)(10 + 490) = 12250.

(ii) Same numbers but including 500: 10, 20, …, 500. 50 terms. Sum = (50/2)(10 + 500) = 12750.

(iii) By inclusion–exclusion: sum of multiples of 2 + sum of multiples of 5 − sum of multiples of 10. Sum of multiples of 2 (1–500): 2 + 4 + … + 500 = 62750. Sum of multiples of 5: 5 + 10 + … + 500 = 25250. Sum of multiples of 10 = 12750. Total = 62750 + 25250 − 12750 = 75250.

Q3. $a_8 = \frac{1}{2} \cdot a_2$ and $a_{11} = (1/3) \cdot a_4 + 1$. Find a_{15}.

Solution

$a + 7d = (a + d)/2 \rightarrow 2a + 14d = a + d \rightarrow a = -13d$. Also $a + 10d = (a + 3d)/3 + 1 \rightarrow 3a + 30d = a + 3d + 3 \rightarrow 2a + 27d = 3$. Substituting $a = -13d$: $-26d + 27d = 3 \rightarrow d = 3$, $a = -39$. So $a_{15} = -39 + 14(3) = 3$.

Answer: $a_{15} = 3$.

Q4. A 37-term AP has the sum of its three middle terms = 225 and sum of its last three = 429. Find the AP.

Solution

Middle three are terms 18, 19, 20; their sum is $3(a + 18d) = 225 \rightarrow a + 18d = 75$. Last three (35, 36, 37): $3(a + 35d) = 429 \rightarrow a + 35d = 143$. Subtracting: $17d = 68 \rightarrow d = 4$, $a = 3$. So the AP is 3, 7, 11, 15, …

Q5. Sum of integers between 100 and 200 that are (i) divisible by 9 (ii) not divisible by 9.

Solution

(i) Multiples of 9 between 100 and 200: 108, 117, …, 198. 11 terms. Sum = $(11/2)(108 + 198) = 11 \times 153 = 1683$.

(ii) Sum of all integers from 101 to 199: $(99/2)(101 + 199) = 14850$. Sum not divisible by 9 = $14850 - 1683 = 13167$.

Q6. If $a_{11} : a_{18} = 2 : 3$, find $a_5 : a_{21}$ and $S_5 : S_{21}$.

Solution

$(a + 10d)/(a + 17d) = 2/3 \rightarrow 3a + 30d = 2a + 34d \rightarrow a = 4d$. Then $a_5 = a + 4d = 8d$; $a_{21} = a + 20d = 24d$. Ratio $8d : 24d = 1 : 3$. For sums: $S_5 = (5/2)(2a + 4d) = (5/2)(12d) = 30d$; $S_{21} = (21/2)(2a + 20d) = (21/2)(28d) = 294d$. Ratio $30 : 294 = 5 : 49$.

Answer: $a_5 : a_{21} = 1 : 3$; $S_5 : S_{21} = 5 : 49$.

Q7. Show that the sum of an AP with first term a, second term b and last term c equals $(a + c)(b + c - 2a) / [2(b - a)]$.

Solution

The common difference is $d = b - a$. Number of terms: $c = a + (n - 1)d \rightarrow n - 1 = (c - a)/(b - a) \rightarrow n = (c - a + b - a)/(b - a) = (b + c - 2a)/(b - a)$. Sum = $(n/2)(a + c) = (a + c)(b + c - 2a)/[2(b - a)]$. ✓

Q8. Solve $-4 + (-1) + 2 + \ldots + x = 437$.

Solution

$a = -4$, $d = 3$. Last term $x = -4 + (n - 1)(3) = 3n - 7$. Sum = $(n/2)(-8 + 3(n - 1)) = n(3n - 11)/2 = 437 \rightarrow 3n^2 - 11n - 874 = 0 \rightarrow (n - 19)(3n + 46) = 0 \rightarrow n = 19$. So $x = 3(19) - 7 = 50$.

Answer: $x = 50$.

Q9. Jaspal repays a loan of Rs 118000 starting with Rs 1000 in month 1, increasing by Rs 100 each month. What is the 30th instalment? What is the balance after 30 months?

Solution

30th instalment = $1000 + 29(100)$ = Rs 3900. Total paid in 30 months = $(30/2)(2000 + 29(100)) = 15(4900)$ = Rs 73500. Balance = $118000 - 73500$ = Rs 44500.

Answer: 30th instalment = Rs 3900; balance after 30 instalments = Rs 44500.

Q10. 27 flags are placed at 2 m intervals along a passage. Flags are stored at the middle flag's position. Ruchi carries one flag at a time, places it, and returns. Find the total distance she walks and the maximum distance she carries any single flag.

Solution

With 27 flags, the middle one is the 14th, and there are 13 flags on each side at distances 2, 4, 6, …, 26 metres from the centre. For each side: each round trip is $2 \times$ distance. Sum on one side $= 2 \times (2 + 4 + \ldots + 26) = 2 \times 182 = 364$ m. Both sides together: $2 \times 364 = 728$ m. The farthest flag (on either side) is 26 m away, so the maximum distance she carries one flag is 26 m.

Answer: **Total distance = 728 m; maximum distance carrying a flag = 26 m.**

Quick Revision Sheet (Night Before the Exam)

- $a_n = a + (n - 1)d$ — nth term formula.
- $S_n = (n/2)[2a + (n - 1)d]$ or $S_n = (n/2)(a + l)$.
- $a_n = S_n - S_{n-1}$ — useful when S_n is given as a formula in n.
- $a_m - a_n = (m - n)d$. Differences between corresponding terms of two APs with the same d stay constant.
- If n is odd, sum = n × middle term.
- Three terms in AP: write as a − d, a, a + d. Four terms: a − 3d, a − d, a + d, a + 3d. These symmetric setups simplify sum-and-product problems.
- Multiples of LCM problems: 'multiples of both 2 and 5' means multiples of 10, etc.
- Story problems involving repeated tasks at increasing distances (flag problem): each round trip is twice the distance, and you sum the round-trips.
- For 'first negative term', solve $a_n < 0$ for n and take the smallest integer satisfying it.
- 'a_n is a linear polynomial in n' test: if your candidate for a_n is not of the form (linear in n), it cannot be an AP.